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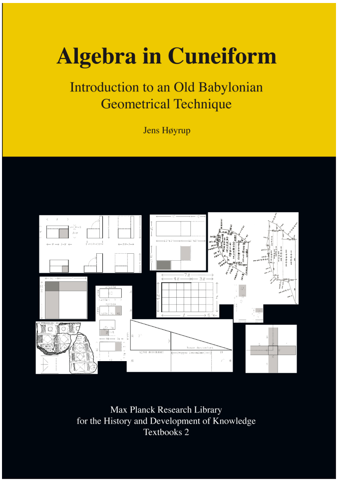
Jens Høyrup:

General Characteristics



In: Jens Høyrup: *Algebra in Cuneiform : Introduction to an Old Babylonian Geometrical Technique*
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Chapter 6
General Characteristics

Drawings?

All the texts that were discussed above were illustrated by geometric drawings. However, only two of the tablets carried geometric diagrams, and in both cases these illustrated the problem statement, not the procedure.

Many aspects of the procedures are inexplicable in the traditional arithmetical interpretation but naturally explained in a geometrical reading. In consequence, *some* kind of geometry must have participated in the reasoning of the Babylonians. It is not very plausible, however, that the Babylonians made use of drawings quite like ours. On the contrary, many texts give us reasons to believe that they were satisfied with rudimentary structure diagrams; see for example page 52 on the change of scale in one direction. The absence of particular names for $L = 3\lambda$ and $W = 21\phi$ in TMS IX #3 (see page 59) also suggests that no new diagram was created in which they could be identified, while λ and ϕ *could* be identified as sides of the "surface 2."

After all, that is no wonder. Whoever is familiar with the Old Babylonian techniques will need nothing but a rough sketch in order to follow the reasoning; there is not even any need to perform the divisions and displacements, the drawing of the rectangle alone allows one to grasp the procedure to be used. In the same way as we may perform a mental computation, making at most notes for one or two intermediate results, we may also become familiar with "mental geometry," at most assisted by a rough diagram.

A fair number of field plans made by Mesopotamian scribes have survived; the left part of Figure 6.1 shows one of them. They have precisely the character of structure diagrams; they do not aim at being faithful in the rendering of linear proportions, as will be seen if we compare with the version in correct proportions to the right. In that respect they are similar to Figure 4.5, whose true proportions can be seen in Figure 4.6—pages 65 and 68, respectively. Nor are they interested in showing angles correctly, apart from the "practically right" angles that serve area calculations and therefore have a *structural* role.

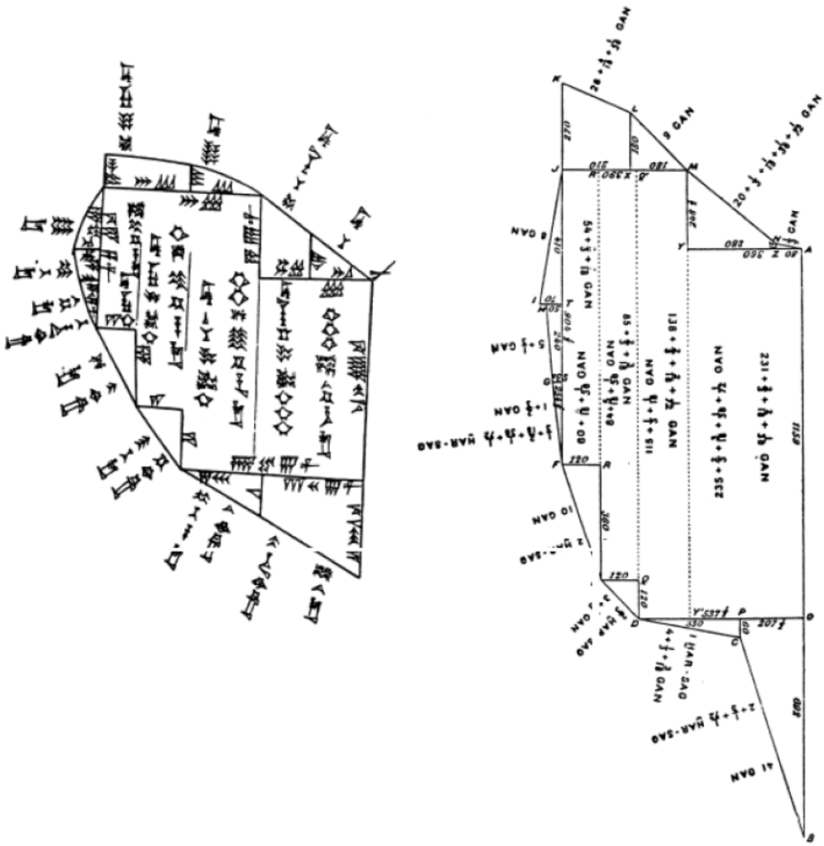

Figure 6.1: A neo-Sumerian field plan (21st century BCE), left as drawn on the tablet, right redrawn in correct proportions. After F. Thureau-Dangin, "Un cadastre chaldéen". *Revue d'Assyriologie* **4** (1897–98), 13–27.

Practicing "mental geometry" presupposes that one has first trained concrete geometry; real drawings of some kind must thus have existed. However, cut-and-paste operations are not easily made on a clay tablet. The dust abacus, used by Phoenician calculators in the first millennium BCE and then taken over by Greek geometers,[1] is much more convenient for this purpose. Here it is easy to cancel

[1] The Greek word for the abacus, αβαξ, is borrowed from a Phoenician root from which come words for "dust" and "flying away."

a part of a figure and to redraw it in a new position. A school-yard strewn with sand (cf. page 33) would also be convenient.

In the same way, dust or sand appears to have served in the first steps of learning the script. From this initial phase, we know the tablets on which are inscribed *the models* the students are supposed to have reproduced in order to learn the cuneiform characters. From *the next* phase we also have the clay tablets written by the students—but from the first phase the work of students has left no archaeological traces, which means that these will probably have been drawn in sand or dust. There is therefore no reason to be astonished that the geometrical drawings from the teaching of algebra and quasi-algebra have not been found.

Algebra?

Until now, for reasons of convenience and in agreement with the majority of historians of mathematics, we have spoken of an Old Babylonian "algebra" without settling the meaning one should ascribe to this modern word in a Babylonian context, and without trying to explain why (or whether) a geometrical technique can really be considered an "algebra."

On our way, however, we have accumulated a number of observations that may help us form a reasoned opinion (at times hinting at the role these observations are going to play in the argument).

At first it must be said that the modern algebra to which the Old Babylonian technique might perhaps be assimilated is precisely *a technique*, namely the practice of equations. Nothing in the Old Babylonian texts allows us to assume that the Babylonians possessed the slightest hint of something like the algebraic *theory* which has developed from the sixteenth century (concerning the link between coefficients and roots, etc.)—nor *a fortiori* to equate what they did with what professional mathematicians today call algebra (group theory and everything building on or extending that domain). The algebra of today which we should think of is what is learned in school and expressed in equations.

We have seen above (page 29) the sense in which the Old Babylonian problem statements can be understood as *equations*: they may indicate the total measure of a combination of magnitudes (often but not always geometric magnitudes); they may declare that the measure of one combination equals that of another one; or that the former exceeds or falls short of the latter by a specified amount. The principle does not differ from that of any applied algebra, and thus not from the equations with which an engineer or an economist operates today. In this sense, the Old Babylonian problem statements are true equations.

But there is a difference. Today's engineer *operates on* his equations: the magnitudes he moves from right to left, the coefficients he multiplies, the functions he integrates, etc.—all of these exist only as elements of the equation and have no other representation. The operations of the Babylonians, on the contrary, were realized within a *different* representation, that of measured geometric quantities.[2]

With few exceptions (of which we have encountered none above) the Old Babylonian solutions are *analytic*. That also makes them similar to our modern equation algebra. Beyond that, most of their procedures are "homomorphic" though not "isomorphic" analogues of ours, or at least easily explained in terms of modern algebra.

These shared characteristics—statements shaped as equations, analysis, homomorphic procedures—have induced many historians of mathematics to speak of a "Babylonian algebra" (*seduced*, certain critics have said during the last 40 years). But there is a further reason for this characterization, a reason that may be more decisive although it has mostly gone unnoticed.

Today's equation algebra possesses a neutral "fundamental representation" (see page 16): abstract numbers. This neutral representation is an empty container that can receive all kinds of measurable quantities: distances, areas, electric charges and currents, population fertilities, etc. Greek geometric analysis, on the other hand, concerns nothing but the geometric magnitudes it deals with, these represent nothing but what they are.

In this respect, the Babylonian technique is hence closer to modern equation algebra than is Greek analysis. As we have seen, its line segments may represent areas, prices (better, inverse prices)—and in other texts numbers of workers and the number of days they work, and the like. We might believe (because we are habituated to confound the abstract geometric plan and the paper on which we draw) that geometry is less neutral than abstract numbers—we know perfectly well to distinguish the abstract number 3 from 3 pebbles but tend to take a nicely drawn triangle for the triangle itself. But even if we stay in our confusion we must admit that from the *functional point of view*, the Old Babylonian geometry of measured magnitudes is also an empty container.

Today's equation algebra is thus a *technique to find* by means of the *fiction that we have already found* (analysis) followed by the manipulation of unknown magnitudes as if they were known—everything within a representation that is functionally empty (namely, the realm of abstract numbers). Replacing *numbers* with *measurable geometric quantities* we may say the same about the Old Baby-

[2]Only first-degree transformations like those of TMS XVI #1 and TMS IX #3 may be seen as constituting a partial exception; TMS XVI #1 is indeed an explanation of how operations directly on the words of the equation are to be understood in terms of the geometric representation. Once that had been understood, TMS IX #3 could probably operate directly on the level of words. But TMS XVI #1 is no problem solution, and in TMS IX #3 the first-degree transformation is subordinate to geometric operations.

lonian technique—with a small reserve to which we shall return presently. If the modern technique is understood as an "algebra" in spite of its immense conceptual distance from group theory and its descendants, it seems reasonable to classify the Old Babylonian technique as we have encountered it in Chapters 2–4 under the same heading.

That does not mean that there are no differences; there are, and even important differences; but these are not of a kind that would normally be used to separate "algebra" from what is not algebra.

Apart from the representation by a geometry of measurable magnitudes, the most important difference is probably that Old Babylonian second- (and higher-) degree algebra had no practical application—not because it could not have for reasons of principle (it could quite well) but because no practical problem within the horizon of an Old Babylonian working scribe asked for the application of higher algebra. All problems beyond the first degree are therefore artificial, and all are constructed backwards from a known solution (many first-degree problems are so, too). For example, the author begins with a square of side $10'$ and then finds that the sum of the four sides and the area is $41'40''$. *The problem* which he constructs then states this value and requires (with a formulation that was in favor among the calculators of the Middle Ages but which is also present in TMS XVI and TMS VII) that the sides and the area be "separated" or "scattered."[3]

This kind of algebra is very familiar today. It allows teachers and textbook authors to construct problems for school students for which they may be sure of the existence of a reasonable solution. The difference is that *our* artificial problems are supposed to train students in techniques that will later serve in "real-life" contexts.

What we do not know is the candor with which certain Old Babylonian texts speak of the value of magnitudes that in principle are supposed not to be known. However, since the text distinguishes clearly between *given* and *merely known* magnitudes, using the latter only for identification and pedagogical explanation, this seemingly deviating habit first of all illustrates the need for a language in which to describe the procedure—an alternative to the ℓ, λ and L of our algebra and the "segment AB" of our geometry. Since the texts represent the "teacher's manual" (notwithstanding the "you" that pretends to address the student), we cannot exclude that the true oral exposition to students would instead make use of a finger pointing to the diagram ("this width here," "that surface there"). Nor can we claim that things will really have occurred like that—we have no better

[3]See TMS XVI #2 line 16 and TMS VII #1 line 4 (below, pages 117 and 118); the two terms seem to be synonymous. This "separation" or "dispersion," which is no subtraction, is the inverse operation of "heaping."

window to the didactical practice of Old Babylonian mathematics than what is offered by TMS XVI #1 (page 27).