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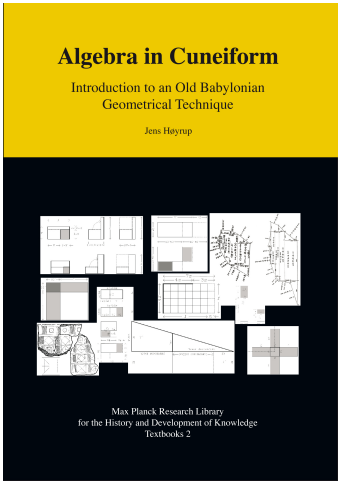
Jens Høyrup:

Appendix A: Problems for the Reader



In: Jens Høyrup: *Algebra in Cuneiform : Introduction to an Old Babylonian Geometrical Technique*
Online version at `http://mprl-series.mpg.de/textbooks/2/`

ISBN 978-3-945561-15-7
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Printed and distributed by:
PRO BUSINESS digital printing Deutschland GmbH, Berlin
`http://www.book-on-demand.de/shop/15336`

The Deutsche Nationalbibliothek lists this publication in the Deutsche Nationalbibliografie; detailed bibliographic data are available in the Internet at `http://dnb.d-nb.de`

Appendix A: Problems for the Reader

The problems presented in Chapters 2–5 were so different one from the others that it was necessary to accompany each of them with a copious commentary. In order to allow the reader who may like to explore some Old Babylonian texts without being held firmly by the hand, this appendix contains problems in translation only, or at most accompanied by the most necessary clarifications. Some are counterparts of problems that were presented above and come from the same tablets.

TMS XVI #2

13. The 4th of the width to that by which the length goes beyond the width, to join,
14. 15′. You, 15′ to 4 raise, 1 you see, what is it?
15. 4 and 1 posit.
16. 15′ scatter. 10′, the going-beyond, and 5′, the joined, posit. 20′, the width,
17. to 10′, the going-beyond, join, 30′ the length, and 20′, to tear out, posit. 5′ to 4 raise,
18. 20′ you see. 20′, the width, to 4 raise, 1°20′ you see.
19. 30′, the length, to 4 raise, 2 you see. 20′, the width,
20. from 1°20′ tear out, 1 you see. 1
21. from 2, the lengths, tear out, 1 you see, what is it?
22. From 4, of the fourth, 1 tear out, 3 you see. IGI 4 detach, 15′ you see.
23. 15′ to 3 raise, 45′ you see, as as much as (there is) of widths posit. Posit to tear out.
24. 1 as as much as (there is) of lengths posit. [...] 1 take, to 1 length
25. raise, 30′ you see. 20′ the width, 20′ to 45′, (as much as (there is) of) widths, raise,
26. 15′ you see, 15′ to 15′ join, 30′ you see, 30 the length.

Commentary: see #1 of the same tablet, page 27.

TMS VII #1

1. The 4th of the width to the length I have joined, its 7⟨th⟩ to 10 I have gone,
2. as much as the heap of length and ⟨width⟩. You, 4 posit; 7 posit;
3. 10 posit; 5′ to 7 raise, 35′ you see.
4. 30′ and 5′ single out. 5′, the step, to 10 raise,
5. 50′ you see. 30′ and 20′, posit. 5′, the step, to 4, of the fourth of the width,
6. raise: 20′ you see, 20′, the width. 30′ to 4, of the fourth,
7. raise, 2 you see. 2 posit, lengths. 20′ from 20′ tear out,
8. and from 2, 30′ tear out, 1°30′ you see.
9. From 4, of the fourth, 1 tear out, 3 {…} you see.
10. IGI 3 detach, 20′ you see. 20′ to 1°30′ raise:
11. 30′ you see, 30′ the length. 30′ from 50′ tear out, 20′ you see, 20′ the width.
12. Turn back. 7 to 4, of the fourth, raise, 28 you see.
13. 10 from 28 tear out, 18 you see. IGI 3 detach,
14. 20′ you see. 20′ to 18 raise, 6 you see, 6 (for) the length.
15. 6 from 10 tear out, 4 (for) the width. 5′ to 6 raise,
16. 30′ the length. 5′ to 4 raise, 20′ you see, 20′ the ⟨width ⟩.

Commentary: see #2 of the same tablet, page 34.

VAT 8389 #1

Obv. I

1. From 1 BÙR 4 GUR of grain I have collected,
2. from 1 second BÙR 3 GUR of grain I have collected.
3. grain over grain, 8‘20 it went beyond
4. My plots I have accumulated: 30‘.
5. My plots what?
6. 30‘, the BÙR, posit. 20‘, the grain which he has collected, posit.
7. 30‘, the second BÙR, posit.
8. 15‘, the grain which he has collected,
9. 8‘20 which the grain over the grain went beyond,
10. and 30‘ the accumulation of the surfaces of the plots posit:
11. 30‘ the accumulation of the surfaces of the plots
12. to two break: 15‘.
13. 15‘ and 15‘ until twice posit:
14. IGI 30‘, of the BÙR, detach: 2″.
15. 2″ to 20‘, the grain which he has collected,

16. raise, 40′ the false grain; to 15‘ which until twice
16a. you have posited,
17. raise, 10‘ may your head hold!
18. IGI 30, of the second BÙR, detach, 2″.
19. 2″ to 15‘, the grain which he has collected,
20. raise, 30′ the false grain; to 15 which until twice
20a. you have posited, raise, 7‘30.
21. 10‘ which your head holds
22. over 7‘30 what goes beyond? 2‘30 it goes beyond.
23. 2‘30 which it goes beyond, from 8‘20
24. which the grain over the grain goes beyond,

Obv. II

1. tear out: 5‘50 you leave.
2. 5‘50 which you have left
3. may your head hold!
4. 40′, the change, and 30′, the change,
5. accumulate: 1°10′. The IGI I do not know.
6. What to 1°10′ may I posit
7. which 5‘50 which your head holds gives me?
8. 5‘ posit. 5‘ to 1°10 raise.
9. 5‘50 it gives to you.
10. 5‘ which you have posited, from 15‘ which until twice
11. you have posited, from one tear out,
12. to one join:
13. The first is 20‘, the second is 10‘.
14. 20‘ (is) the surface of the first plot, 10‘ (is) the surface of the second plot.
15. If 20‘ (is) the surface of the first plot,
16. 10‘ the surface of the second plot, their grains what?
17. IGI 30‘, of the BÙR, detach: 2″.
18. 2″ to 20‘, the grain which he has collected,
19. raise, 40′. To 20‘, the surface of the first plot,
20. raise, 13‘20 the grain of 20‘, the surface of the plot.
21. IGI 30‘, of the second BÙR, detach: 2″.
22. 2″ to 15‘, the grain which he has collected, raise, 30′.
23. 30′ to 10‘, the surface of the second plot
24. raise, 5 the grain of the surface of the second plot.
25. 13‘30 the grain of ⸢the surface⸣ of the first plot
26. over 5 the grain of ⸢the surface⸣ of the second plot
27. what goes beyond? 8‘20 it goes beyond.

This problem belongs to one of two twin tablets, containing a total of ten problems about the rent paid for two parcels of a field. On one parcel the rent is 4 GUR of grain per BÙR, on the other it is 3 GUR per BÙR. The present problem informs us also that the total area is 30‘ (SAR = 1 BÙR), and that the difference between the total rents of the two parcels is 8‘20 (SÌLA). The other problems give, for instance, the two areas, or the difference between the areas together with the total rent.

As explained on page 17, the BÙR and the GUR are units belonging to practical life. In order to work in the place-value system we need to convert them into the standard units SAR and SÌLA (1 BÙR = 30‘ SAR, 1 GUR = 5‘ SÌLA); as we see, the difference between the two rents is already given in SÌLA, and the total area in SAR.

A modern reader may find it strange that the two rents per BÙR, which in lines I. 1–2 are given in GUR (per BÙR), are translated into SÌLA in lines I. 6–7 without multiplication; in general, as we see, the text skips no intermediate step. The explanation is that the conversion is made by means of a “metrological table” (probably a table learned by heart). Precisely because such conversions had to be made so often, scribes had tables which not only stated the converted values of the practical units but also of their multiples. However, they had no tables for combined conversions, and therefore the final conversion into SÌLA per SAR asks for calculation.

The modern reader may also wonder that the text does not indicate once for all the value of the BÙR in SÌLA and its IGI. Once more the reason is that the text describes the Old Babylonian calculational technique: the calculator writes on a small tablet for rough work the three numbers 20 (20‘ SÌLA per BÙR), 30 (30“ SAR per BÙR) and 2 (2″, IGI 30‘)—and afterwards, by means of the multiplication table, the product 40 (20‘·2″ = 40′ SÌLA per SAR).

A small explanation may be necessary in order to facilitate understanding of the procedure: first the text determines what the difference between the two rents would be if the two parcels had been equal in area, that is, 15‘ SAR each. This difference is not large enough—it is 2‘30 SÌLA, 5‘50 SÌLA too small—and therefore the first parcel must be enlarged. Each time a SAR is transferred from the second to the first parcel, the difference grows by 40′+30′ SÌLA (the two “changes” of II. 4[1]); the number of SAR that must be transferred is then found by division.

At the end we find a numerical verification. Such verifications are not rare in the Old Babylonian texts even though their presence is not a general norm.

[1] The tablet is damaged at this point, but the traces of signs that remain could well come from the word *takkirtum*, which means “change” or “modification” but does not occur in other mathematical texts. In any case, this philological doubt does not touch the interpretation of the mathematical procedure.

VAT 8390 #1

Obv. I

1. Length and width I have made hold: 10‵ the surface.
2. The length to itself I have made hold:
3. a surface I have built.
4. So much as the length over the width went beyond
5. I have made hold, to 9 I have repeated:
6. as much as that surface which the length by itself
7. was made hold.
8. The length and the width what?
9. 10‵ the surface posit,
10. and 9 (to) which he has repeated posit:
11. The equalside of 9 (to) which he has repeated what? 3.
12. 3 to the length posit
13. 3 to the width posit.
14. Since "so much as the length over the width went beyond
15. I have made hold," he has said
16. 1 from 3 which to the width you have posited
17. tear out: 2 you leave.
18. 2 which you have left to the width posit.
19. 3 which to the length you have posited
20. to 2 which ⟨to⟩ the width you have posited raise, 6.
21. IGI 6 detach: 10′.
22. 10′ to 10‵ the surface raise, 1‵40.
23. The equalside of 1‵40 what? 10.

Obv. II

1. 10 to 3 which to the length you have posited
2. raise, 30 the length.
3. 10 to 2 which to the width you have posited
4. raise, 20 the width.
5. If 30 the length, 20 the width,
6. the surface what?
7. 30 the length to 20 the width raise, 10‵ the surface.
8. 30 the length together with 30 make hold: 15‵.
9. 30 the length over 20 the width what goes beyond? 10 it goes beyond.
10. 10 together with 10 make hold: 1‵40.
11. 1‵40 to 9 repeat: 15‵ the surface.
12. 15‵ the surface, as much as 15‵ the surface which the length
13. by itself was made hold.

As support for the interpretation, a diagram may serve (Figure 10.1). Then the text almost explains itself, in particular if one keeps in mind BM 13901 #10 (page 46) and BM 15285 #24 (page 93).

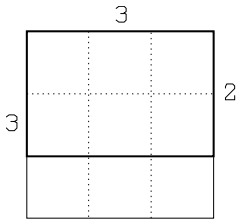


Figure 10.1: The geometry of VAT 8390 #1.

One should take note of the use of the multiplicative operations "make hold," "raise" and "repeat." That "making hold" really implies a *construction* is underlined in I. 3, as we have also seen in AO 8862 #2 (page 60). The "raising" in I. 20 and II.7 is of special interest: it finds the area of rectangles, but as these are already in place, there is no need to construct them. Therefore the area is merely *calculated*.

VAT 8520 #1

Obv.

1. The 13th from the heap of the *igûm* and the *igibûm*
2. to 6 I have repeated, from the inside of the *igûm*
3. I have torn out: 30′ I have left. 1 the surface. The *igûm* and the *igibûm* what?
4. Since "the thirteenth of the heap of the *igûm* and the *igibûm*
5. to 6 I have repeated, from the inside of the *igûm*
6. I have torn out: 30′ I have left," he has said,
7. 13, of the thirteenth, posit; 6 to which he has repeated posit;
8. 1, the surface, posit; and 30′ which he has left posit.
9. From 13, of the thirteenth, 6 to which he has repeated
10. tear out. 7 you leave.
11. 7 which you leave and 6 to which you have repeated,
12. may your head hold!
13. 7 to 6 raise, 42 to 1, the surface, raise, 42.
14. 42, may your head hold!
15. 13, of the thirteenth, to 30′ which he has left

16. raise, 6°30′ to two break: 3°15′.
17. 3°15′ together with 3°15′ make hold: 10°33′45″.
18. To 10°33′45″, 42 which your head holds
19. join, 52°33′45″.
20. The equal of 52°33′45″ what? 7°15′.
21. 7°15′ and 7°15′, its counterpart, lay down:
22. 3°15′, the made-hold, from one tear out, to the other join:
23. The first is 10°30, the other is 4.
24. What to 7, which your head holds, should I posit
25. which 10°30′ gives me? 1°30′ posit. 1°30′ to 7 raise,
26. 10°30′ it gives you. 1°30′ which you have posited is the *igûm*.
27. IGI 6, which your head holds, detach, 10′.
28. 10′ to 4 raise, 40′ is the *igibûm*.
29. Since 1°30′ is the *igûm*, 40′ is the *igibûm*, the surface is what?
30. 1°30′, the *igûm*, to 40′, the *igibûm*, raise, 1 is the surface.
31. 1°30, the *igûm*, and 40′, the *igibûm*, heap: 2°10′.

Rev.

1. The thirteenth of 2°10′ what? 10′.
2. 10′ to 6 repeat: 1, from 1°30,
3. the *igûm*, tear out: 30′ you leave.

Like YBC 6967 (page 46), this problem deals with a number pair from the table of reciprocals. Both texts speak of their product as "the surface," in agreement with the geometric representation. But there is a difference: this time the product is 1, not 1ˈ as in YBC 6967.

As regards the mathematical structure and the procedure, one may compare with TMS IX #3 (page 57).

Str 368

Obv.

1. I have taken a reed, its measure I do not know.
2. 1 KÙŠ I have cut off. 1 sixty (steps along) the length I have gone.
3. (With) what I have cut off I have enlarged it
4. with 30 (steps) of that (along) the width I have gone.
5. 6ˈ15 is the surface. The head (initial length) of the reed what?
6. You, by your proceeding,
7. 1ˈ and 30 posit. (For) the reed which you do not know
8. 1 posit, to 1 sixty which you have gone

9. you raise: 1‵ is the false length.
10. 30 to this 1 raise, 30 is the false width.
11. 30, the false width to 1‵, the false length,
12. raise, 30‵ the false surface.
13. 30‵ to 6‵15, the true surface,

Rev.

1. raise: 3‴7‶30‵ it gives you.
2. 5′ which you have cut off to the false length raise,
3. 5 it gives you. 5 to the false width raise,
4. 2‵30 it gives you. $\frac{1}{2}$ of 2‵30 break, 1‵15
5. 1‵15 make encounter, 1‶33‵45
6. to 3‴7‶30‵ join, 3‴9‶3‵45.
7. What is equal? 13‵45 is equal.
8. 1‵15 which you have made encounter to the inside join,
9. 15‵ it gives you. IGI 30‵, the false surface, detach, 2″.
10. 2″ to 15‵ raise, 30′ is the head of the reed.

This is the rectangle version of the "broken reed" (see page 70), similar to VAT 7532. In this variant, the field is rectangular, and the reed breaks a single time only.

YBC 6504 #1

Obv.

1. So much as length over width goes beyond, I have made confront itself, from the inside of the surface
2. I have torn it out: 8′20″. Length over width 10′ goes beyond.
3. By your proceeding, 10′ you make hold:
4. 1′40″ to 8′20″ you join: 10′ you posit.
5. Half of 10′ you break: 5′ you posit.
6. 5′ you make hold: 25″ you posit.
7. 25″, the surface, to 10′ you join: 10′25″ you posit.
8. By 10′25″, 25′ is equal. 5′ to 25′ you join:
9. 30′, the length, you posit. 5′ from 25′ your tear out:
10. 20′, the width, you posit.

This problem deals with the same mutilated rectangle as #4 of the same tablet (see page 79): Together, indeed, the four problems of the tablet represent an interesting variant of the closed group where the "surface" of a rectangle is given together with the length; with the width; with the sum of the sides; or with their

difference (see note 3, page 108). In the present tablet, the "surface" is replaced everywhere by the same mutilated rectangle.

In this first problem, we know the side of the square that has been "torn out." It is therefore easily reduced to the type we know from YBC 6967 (page 46). In following the operations one should keep in mind that the number $10'$ occurs in two different roles.

Exceptionally in this type, the "joining" of $5'$ precedes the "tearing out." The tablet seems to belong to the same early phase and text group as AO 8862 , and it shares this particularity with three texts from Eshnunna (thus belonging to an even earlier phase). It seems indeed that the school is responsible for the request that operations should always be concretely meaningful, just as it was responsible for outlawing broad lines—this request is not evidence of "a primitive intellect not yet ready for abstraction," as has been supposed, but of a critical mind reflecting upon how to justify what is done.

YBC 6504 #3

Rev.

1. So much as length over ⟨width⟩ goes beyond, made encounter, from inside the surface I have torn out,
2. $8'20''$. $30'$ the length, its width what?
3. $30'$ made encounter: $15'$ you posit.
4. $8'20''$ from inside $15'$ you tear out, $6'40''$ you posit.
5. Half of $30'$ you break:
6. $15'$ made encounter: $3'45''$ you posit.
7. $3'45''$ to $6'40''$ you join: $10'25''$ you posit.
8. By $10'25''$, $25'$ is equal. $15'$ from $25'$ you tear out:
9. $10'$ you posit. $10'$ from $30'$ you tear out:
10. $20'$, the width, you posit.

This is the third problem from the same tablet. It makes use of a ruse which is both elegant and far from every routine (see Figure 10.2): elimination of the mutilated rectangle from the square $\square(\ell)$ on the length leaves a remainder that can be decomposed as a square $\square(\ell - w)$ and a rectangle $\sqsubset\sqsupset(\ell - w, 30')$. These can be reconfigured as a gnomon, as shown in the diagram. We may look at the process as a "change of variable"—the problem now concerns a square $\square(\ell - w)$ and 30 of its sides, and its solution follows the book for such problems.

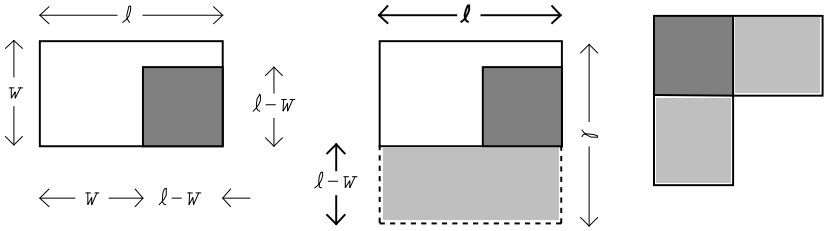


Figure 10.2: The geometry behind YBC 6504 #3, in slightly distorted proportions.

BM 85200+VAT 6599 #23

Rev. I

19. An excavation. So much as I have made confront itself, and 1 kùš, going beyond, that is the depth. 1°45′ of dirt I have torn out.
20. You, 5′, going beyond, to 1, the conversion, raise, 5′ you see; to 12 raise, 1 you see.
21. 5′ make confront itself, 25″ you see. 25″ to 1 raise, 25″ you see. igi 25 detach,
22. 2‘24 you see. 2‘24 to 1°45′ raise, 4‘12 you see.
23. from “equal, 1 joined,” 6 ⸢1⸣ is/are equal(s). 6 to 5′ raise, 30′ you see, confronts itself. 6 (error for 7) the depth.
24. The procedure.

This problem comes from the same tablet as the “excavation problem” BM 85200+VAT 6599 #6 that was dealt with above (page 89), and its solution follows the same principles. Now the “ground” is square, and the depth exceeds the side by 1 kùš. As “reference body” a cube of side 1 kùš is chosen, which allows the use of a table of $n^2 \cdot (n+1)$, called “equal, 1 joined.” Such tables have been found.

Db_2–146

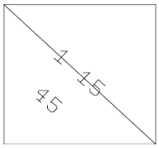
Obv.

1. If, about a (rectangle with) diagonal, (somebody) asks you
2. thus, 1°15 the diagonal, 45′ the surface;
3. length and width corresponding to what? You, by your proceeding,
4. 1°15′, your diagonal, its counterpart lay down:
5. make them hold: 1°33′45″ comes up,

6. 1°33′45″ ˹may your˺ hand ˹hold˺
7. 45′ your surface to two bring: 1°30′ comes up.
8. From 1°33′45″ cut off: {…} 3′45″ the remainder.
9. The equal of 3′45″ take: 15′ comes up. Its half-part,
10. 7′30″ comes up, to 7′30″ raise: 56″15‴ comes up
11. 56″15‴ your hand. 45′ your surface over your hand,
12. 45′56″15‴ comes up. The equal of 45′56″15‴ take:
13. 52′30″ comes up, 52′30″ its counterpart lay down,
14. 7′30″ which you have made hold to one
15. join: from one
16. cut off. 1 your length, 45 the width. If 1 the length,
17. 45 the width, the surface and the diagonal corresponding to what?
18. You, by your making, the length make hold:
19. 1 comes up … may your head hold.

Rev.

20. … : 45′, the width, make hold:
21. 33′45″ comes up. To your length join:
22. 1°33′45″ comes up. The equal of 1°33′45″ take:
23. 1°15′ comes up. 1°15′ your diagonal. Your length
24. to the width raise, 45′ your surface.
25. Thus the procedure.



This is one of the texts from the Eshnunna region, and thus belongs to the earliest phase (and as we see, it uses the phrase "to one join, from one cut off," not respecting the "norm of concreteness"). With fair precision it can be dated to c. 1775 BCE. The problem is one of the riddles which the Old Babylonian school borrowed from the Akkadian surveyors (see pages 106 and 107); it turns up, solved in precisely the same way, in a Hebrew manual from 1116 CE, that is, 1900 years later. In the text we see several reminiscences of this origin—for instance the introductory passage "If, about a (rectangle with) diagonal, (somebody) asks you thus" and the reference to the square on the length in line 21 simply as "your length"; both features reverberate in BM 13901 #23.

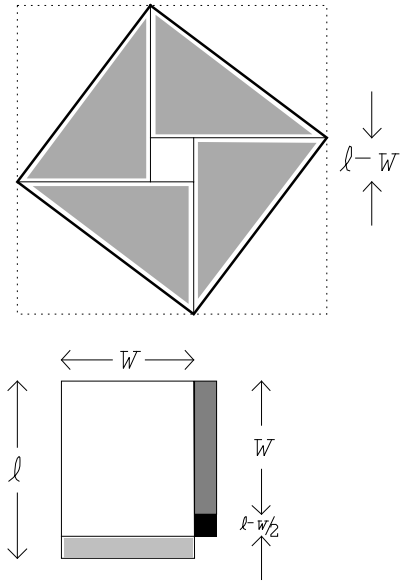


Figure 10.3: The geometry of Db_2-146.

Lines 1–9 find the difference between the length and the width of the rectangle; the method is shown in the upper part of Figure 10.3. Afterwards, the sides are found from this difference and the area by the procedure which we already know perfectly well, for instance from YBC 6967 (see page 46), and which corresponds to the lower diagram in the figure. (However, the use of "raising" in Obv. 10 shows that the procedure is supposed to be supported by the already existing upper diagram.)

The "hand" of lines 6 and 11 is a reference to the reckoning board on which the calculator performed his additions and subtractions. The "half-part" of line 9 (*muttatum*) is a synonym for "moiety."

In the end we have a proof with an unmistakeable trace of the "Pythagorean rule" in abstract formulation (*the length make hold*, without the usual identification of its numerical value).