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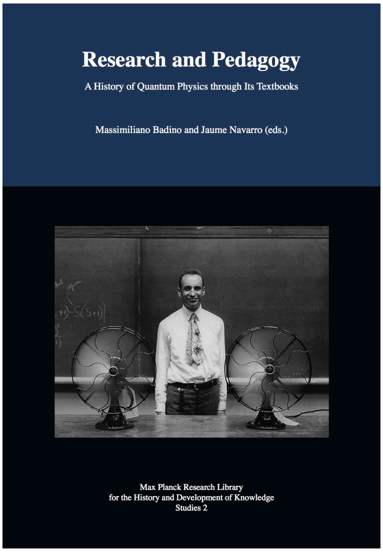
Paul Dirac and *The Principles of Quantum Mechanics*



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Chapter 10
Paul Dirac and *The Principles of Quantum Mechanics*
Helge Kragh

10.1 Paul Dirac and Early Quantum Theory

Although not well known to the general public, Paul Adrien Maurice Dirac hardly needs to be introduced to physicists and historians of science. Born in Bristol in 1902 as a Swiss citizen—his father was Swiss and Paul only acquired British nationality in 1919—he became one of the most important theoretical physicists ever. His impact on modern physics may even have been greater than that of Einstein (Zichichi 2000). Young Dirac made his first breakthrough in the fall of 1925 when he developed his own version of quantum mechanics, known as q-number algebra, and over the next few years he established himself as a leading expert in the new quantum physics. In 1927–28 he made pioneering contributions to quantum statistics (Fermi-Dirac statistics), quantum electrodynamics, and relativistic quantum theory. The linear and relativistically invariant wave equation for the electron that he published in early 1928 not only explained the electron's spin and magnetic moment, but also, three years later, led to the prediction of antielectrons (positrons) and antiparticles more generally.

Dirac's genius was recognized early on. For example, he was part of the exclusive company of physicists invited to the famous Solvay conference in 1927. In 1930, at the unusually young age of 27, he was elected a fellow of the prestigious Royal Society, and the same year he published his monumental *Principles of Quantum Mechanics*, the subject of this essay. Two years later he was appointed Lucasian Professor of mathematics at Cambridge University, the chair once held by Isaac Newton and later by Stephen Hawking. Another high point of Dirac's career came in 1933, when he was awarded the Nobel Prize in physics, sharing it with Erwin Schrödinger. Although Dirac's scientific fame is closely linked to his fundamental contributions to quantum theory, and especially to those of the period 1925–34, he also dealt with other subjects, including cosmology, classical electron theory, and the general theory of relativity. Moreover, the influence of his ideas extended beyond physics, especially to mathematics (cp. the Dirac δ-function, Dirac matrices, and Dirac operators). Paul Dirac remained Lucasian Professor until his retirement in 1969, when he joined the physics department of Florida State University in Tallahassee. He died in 1984, and in 1995 a commemorative stone carrying his name and equation was unveiled at a ceremony in Westminster Abbey.[1]

[1] On Dirac's life and science, see (Dalitz and Peierls 1986; Kursunoglu and Wigner 1987; Kragh 1990). Dirac's private life is covered in detail in (Farmelo 2009). On Dirac as Lucasian Professor, see (Kragh 2003). Other secondary sources can be found in these works.

10.2 Origin and Dissemination

While still a Ph.D. student, under the supervision of Ralph Fowler, Dirac was assigned to lecture on the new and exciting developments in quantum theory. This first course ever on quantum mechanics at a British university was given in the Easter term of 1926 and attended by, among others, Nevill Mott, John A. Gaunt, Alan H. Wilson, Douglas Hartree, William McCrea, and Julius Robert Oppenheimer. Also Fowler and some of his students joined the course. McCrea recalled that the material of the lectures was close to that later presented in *Principles*, if, of course, restricted to what was known at the time (McCrea 1985; 1986).[2] The following year Dirac started giving a regular course on quantum mechanics, which he would continue to do until the 1960s.

The content of Dirac's early lectures formed the basis of the textbook that appeared in the summer of 1930, and which he subsequently used for his course. Given the scarcity of suitable textbooks in quantum mechanics at the time, and that Dirac had already prepared extensive lecture notes on the subject, it was natural for him to transform and update these into a proper textbook. Indeed, with respect to both structure and content there is a great deal of similarity between his lecture notes of 1927–28 and his textbook. Moreover, it was important for Dirac to present the principles of the new quantum mechanics in the way *he* thought they should be presented, namely as a concise and coherent symbolic calculus that allowed comparison between calculated quantities and those found experimentally. To him, the new physics was basically a formal scheme that allowed the calculation of experimental results, while it had nothing to say about ontological questions. The proper way of presenting quantum mechanics must necessarily be abstract, he wrote, for the new theory is "built up from physical concepts which cannot be explained in terms of things previously known to the student, which cannot even be explained adequately in words at all" (Dirac 1930). It was this abstract picture of quantum mechanics that *Principles* conveyed to its readers.

The idea of writing a textbook was not Dirac's, but seems to have come from James Gerald Crowther, a science journalist three years older than Dirac. This also accounts for the fact that the book was published by Oxford University Press and not, as would otherwise have been natural, by Cambridge University Press. Crowther, who had joined Oxford University Press in 1924 as representative for scientific and technical books, established a close relationship with physicists at the Cavendish Laboratory, including Ernest Rutherford and Peter Kapitza. For a time he acted as unofficial press agent for the Cavendish, and he would later write its history (Crowther 1974). On Crowther's initiative, the Oxford University Press decided to establish an International Series of Monographs on Physics with Fowler and Kapitza as general editors. The first book in the series was planned to be Dirac's work on quantum mechanics. Crowther recalled that when he first approached Dirac with the book proposal, "he was living in a simply furnished attic in St. John's College. He had a wooden desk of the kind which is used in schools. He was seated at this, apparently writing the great work straight off" (Crowther 1970, 39).[3]

[2] The content of Dirac's course in 1927–28 appears in his notes for "Lectures on Modern Quantum Mechanics," (AHQP).

[3] On the relationship between Crowther and Dirac, see (Farmelo 2009). The Oxford book series came to include several important monographs on physics, from the 1930s and later. Early examples are George Gamow, *Constitution of Atomic Nuclei and Radioactivity* (1931), John H. Van Vleck, *The Theory of Electric and Magnetic Susceptibilities* (1932), and Richard C. Tolman, *Relativity, Thermodynamics and Cosmology* (1934). The series continues to this very day, comprising a total of about 150 titles.

It should be kept in mind that there were very few British books on quantum theory at the time. According to a catalogue issued by the British Science Guild, in 1930 there were only fourteen British books on quantum topics, and many of them were translations from German or bound collections of lectures.[4] Although *Principles* was not the first book on quantum theory in Britain, it was one of the first. Dirac started writing the book in 1928, but due to travels and a busy scientific schedule—much of it occupied with the consequences of his new theory of the electron—progress was slow. By February 1930 the galley proofs of the book were ready, and about half a year later it appeared in the bookshops as the first volume in the Oxford International Series of Monographs on Physics. The preface was dated 29 May 1930, and the price was 17 shillings and 6 pence.

Principles of Quantum Mechanics became a great (and probably surprising) success, with the first edition selling about two thousand copies. The translations, and especially those in German and Russian, sold even better. The book quickly established itself as the standard work on quantum mechanics, not only used by students as a textbook but also by many experienced physicists. The mathematician Harish-Chandra recalled that he, while an undergraduate of Allahabad University in India, came across a copy of the 1930 edition in the university's library. "[I] was immediately fascinated by it," he said. "The exposition was so lucid and elegant that it gave me the illusion of having understood most of it and prompted in me a strong desire to devote my life to theoretical physics" (Harish-Chandra 1987, 34).[5] *Principles* came out in a substantially rewritten second edition in 1935, and still later editions appeared in 1947 and 1958, with reprints in 1967, 1971, 1974, and 1984. The third and fourth editions differed from the one of 1935 mainly in Dirac's use of the so-called "bracket" notation that he had developed in 1939 and which makes use of quantum states labeled as, for example, $< a|$ and $|b >$ (called a "bra vector" and a "ket vector") (Dirac 1939; Harish-Chandra 1987, 34).[6]

Principles was an enduring success. Paperback reprints of the fourth edition appeared as late as 1993, and the book is still in demand, eighty years after it was first published. The American physicist Philip Morrison exaggerated when he said that "everybody who had ever looked at books had a copy of Dirac," but as far as physicists were concerned, it may have been close to the truth (Weiner 1972, 131). In this essay, I shall be concerned with the first two editions only, those of 1930 and 1935.

10.3 Translations

Dirac's book on the principles of quantum mechanics was translated into German (1930), French (1931), Russian (1932), and Japanese (1936), and possibly into some other languages as well. The German translation, made by Werner Bloch, was arranged at an early time, as evidenced by a letter from Dirac to his Russian colleague Igor Tamm of January 1929, and it appeared only shortly after the English edition.[7] Dirac, who knew German well, checked the translation.

[4]The total number of physics books was 424, of which 39 were on relativity theory. In 1925 there were only 3 books on quantum topics. See (Williamson 1987, 10).

[5]Harish-Chandra became Dirac's research student in 1945. After having obtained his Ph.D. in 1947 and doing some work in theoretical physics, he moved to the United States, where he changed from physics to mathematics.

[6]On the third and fourth editions, see (Brown 2006).

[7]Dirac to Tamm, 3 January 1929, quoted in (Kragh 1990, 79). The letter is reproduced in full in (Kojevnikov 1993, 18–19). Dirac mentioned to Tamm Hermann Weyl's *Gruppentheorie und Quantenmechanik* (1928), which

With good connections to physicists in the Soviet Union, Dirac was also actively involved in bringing out a Russian translation of his book. In the summer of 1930, while attending a conference in Kharkov, he brought the corrected proof sheets with him and handed them over to the Russian theorist Dmitri Ivanenko, whom he had met two years earlier at another conference in Russia (Kojevnikov 1993, 36; Gorelik and V. Y. Frenkel 1990, 156). Edited by Ivanenko and translated by the young Leningrad physicist Matvei Bronstein, the translation appeared in 1932 as *Printsipy Kvantovoi Mekhaniki*. Although the Russian edition was very successful—it sold three thousand copies in a few months—from an ideological point of view, it was seen as somewhat problematical by Soviet commissars. This is reflected in a preface that the publishing house GTTI added to Dirac's own preface, in which it was said that "The publishing house is fully aware that this work contains many views and statements completely at variance with dialectical materialism." Yet it was argued that the material of the book, "critically mastered, can be used at the front in the struggle for dialectical materialism."[8] At the time Dirac's view of quantum mechanics was close to what would later be called the Copenhagen interpretation, and it may have been this view that the publishing house felt its duty to warn against.

Abstract and mostly concerned with foundational matters, *Principles* had little to say about the many applications of quantum mechanics. To make up for this deficiency, Dirac added, on the request of Ivanenko, an extra chapter in which he covered various approximation methods, such as those developed by Vladimir Fock and Douglas Hartree (the Hartree-Fock approximation). Another way in which the Russian edition differed from the original was that Ivanenko added several appendices and Bronstein a number of footnotes. These additions were made in agreement with Dirac. Moreover, Ivanenko provided a long editorial preface in which he praised the book and compared it to other books on quantum mechanics. His comparison is worth quoting:

> Sommerfeld's supplementary volume *Wellenmechanischer Ergänzungsband* appears as a collection of solutions of a series of particular problems; de Broglie's book *Introduction à l'étude de la mécanique ondulatoire* is only an introduction, devoted mainly to the transition from classical to quantum mechanics; Born and Jordan's *Elementare Quantenmechanik* is an exposition of a deliberately restricted part of the material that is amenable to analysis by a special method (Schrödinger's equation does not appear in the book); finally, Frenkel's *Einführung in die Wellenmechanik* is the most accessible of the books for reading but, like all the others, does not give an exposition of the *system* of quantum mechanics. It is the exposition of the system that Dirac's book gives, truly in the highest form, free from all provincialism, […] In our view, the book that is closest in nature, Weyl's *Gruppentheorie und Quantenmechanik*—highly regarded by Dirac—is significantly inferior to

he praised as "very clearly written and […] far the most connected account of quantum mechanics that has yet appeared." But he also pointed out that Weyl's book was "rather mathematical and therefore not very easy."

[8] The Russian translation of the second edition of *Principles* included a similar warning from the publisher, namely that "P. Dirac […] makes some philosophical and methodological generalizations that contradict the only true scientific method of cognition–dialectical materialism." Full translations of the prefaces of the Russian editions appear in (Dalitz 1995, 471–478).

> Dirac's book, on account of both the superfluous mathematical formalism and the actual style of the exposition.[9] (Dalitz 1995, 473)

The second edition of *Principles* also appeared in a Russian translation, published in 1937 by the same publishing house (GTTI). It was edited by Bronstein and translated by C. Angluski.

In 1930 theoretical physics in Japan was beginning to develop under the leadership of Yoshio Nishina, who had spent most of the 1920s in Europe on an extended stay. Nishina had visited Dirac in 1928 and was his host when he, together with Heisenberg, visited Japan in the summer of 1929. It was also Nishina who translated the lectures of Heisenberg and Dirac and had them published (as *Ryôshiron sho mondai*) in 1931. After the publication of *Principles* Nishina thought of producing a Japanese translation, for which he had secured the rights from Dirac and the Oxford University Press. However, for some years nothing happened, and on Dirac's advice it was agreed to make a translation of the forthcoming second edition instead of the 1930 edition. According to the recollection of Hidehiko Tamaki, "the author himself had told him [Nishina] that the time was right to bring out a Japanese version, and that the second edition was written in a style far easier to comprehend than the first one" (Tamaki 1995, 130; Brown 2006, 385). The Japanese translation was published in 1936, translated by a team of physicists consisting of Nishina, Tamaki, Minoru Kobayashi, and Sin-Itiro Tomonaga. Although no translation of the first edition appeared, much of its content was included in a 1932 book with translations of the lectures that Dirac and Heisenberg had given during their stay in 1929 (Dalitz 1995, 657–658).

10.4 Reviews of Principles

The Principles of Quantum Mechanics was widely reviewed in the physics journals, in almost all cases positively and in some enthusiastically. It was a common feature of the reviews to praise the book for its directness, generality, and completeness. Some found it to be elegant. In a review of the German translation, the young Swiss physicist (and later Nobel laureate) Felix Bloch emphasized the originality and closed nature of *Principles*, only regretting that Dirac did not refer to enough of the original literature (Bloch 1931, 456).[10] It is hardly surprising that Bronstein, the translator of the Russian edition, praised Dirac's book for being "the best exposition of quantum mechanics to have appeared so far." The reader, he said, "will never stub his toes against sham academism and silly pedantry." This quality of plain and direct presentation he contrasted with Weyl's *Gruppentheorie und Quantenmechanik*, which to his mind was unnecessarily mathematical and "marred by pedantry" (Bronstein 1931, 355–358), as quoted in Gorelik and Frenkel (1990, 45–46). On this question Bronstein and Ivanenko were of one mind.

When Pauli, known for his sharp tongue and equally sharp pen, reviewed *Principles* in *Die Naturwissenschaften*, he was unusually positive. He expressed admiration for the book as a whole, which he described as "an indispensable standard work." He included in his

[9] Yakov Frenkel's book appeared in 1932 in an English edition as *Wave Mechanics: Elementary Theory* (1932). Ivanenko also mentioned "a short introduction by Haas" that existed in a Russian translation, a reference to the Austrian physicist Arthur E. Haas's *Materiewellen und Quantenmechanik* (1928).

[10] The element of closed nature or self-consistency seems also to have impressed Paul Ehrenfest, who allegedly found it to be "ein greuliches Buch" that was difficult to understand. "A terrible book—you can't tear it apart," he supposedly said according to the recollections of Adriaan Fokker (Kragh 1990, 79).

praise the abstract and symbolic method on which Dirac based his exposition of quantum mechanics, a method Pauli found to be "greatly elegant and general." However, he also pointed out that the consistent use of the symbolic method had its disadvantages, since it might lead to "a certain danger that the theory will escape from reality" (Pauli 1931, 188). Pauli complained that Dirac's book did not reveal the crucial fact that quantum mechanical measurements require real and solid measuring devices that follow the laws of classical physics; measurements in the atomic and subatomic realm are not processes that merely involve mathematical symbols and formulae. While the classical nature of the measurement apparatus was an important element in Pauli's and Bohr's conception of quantum mechanics, it was not a point appreciated by Dirac.

Oppenheimer reviewed *Principles* for American physicists in *Physical Review*, calling it "astonishingly complete" and "unitary and coherent." He likened it to another, older classic of physics, Josiah Willard Gibbs's *Elementary Principles of Statistical Mechanics*. Like this work, Dirac's book "is clear, with a clarity dangerous for a beginner, deductive, and in its foundations abstract; its argument is predominantly analytical; the virtual contact with experiment is made quite late in the book." Oppenheimer realized that Dirac's text, in spite of all its qualities, was not ideal for a first course in quantum mechanics. "The book remains a difficult book, and one suited only to those who come to it with some familiarity with the theory. It should not be the sole text, nor the first text, in quantum theory, just as that of Gibbs's should not be the first in statistical mechanics" (Oppenheimer 1931, 97).

Dirac's book was reviewed anonymously in *Nature* alongside two other works on quantum theory, Heisenberg's *Die physikalischen Prinzipien der Quantentheorie* and Léon Brillouin's *La théorie des quanta*. The author of the reviews was almost certainly Arthur Eddington, such as revealed by the style and terminology (Eddington 1931, 699). Inspired by Dirac's relativistic wave equation of 1928, Eddington had recently begun his lonely and ambitious attempt to unify the quantum world with the universe at large, a line of work that in 1946 would lead to his posthumously published *Fundamental Theory*. The review in *Nature* focused on Dirac's more general conception of quantum mechanics as a theory that could not be understood in terms of models or classical concepts. Dirac's "logical and original mode of approach" to the problems of quantum theory greatly appealed to Eddington:

> He bids us throw aside preconceived ideas regarding the nature of phenomena and admit the existence of a substratum of which it is impossible to form a picture. We may describe this as the application of "pure thought" to physics, and it is this which makes Dirac's method more profound than that of other writers. […] He introduces a new attitude of mind towards the investigation of Nature, and the interest lies in watching the development of progress of his ideas. There can be no doubt that his work ranks as one of the high achievements of contemporary physics.

Other reviews of Dirac's book appeared in journals not read by the majority of physicists. Heisenberg reviewed it in a weekly magazine on metallurgy, the *Metallwirtschaft*, pointing out that practical applications were given much less priority than the general principles and only included to further the understanding of the latter. Like Pauli, he had his reservations with regard to the consistent use of what Dirac called the symbolic method. He had the impression, he wrote, "that perhaps Dirac presents quantum mechanics, and especially its physical content, as somewhat more 'symbolic' than is necessary" (Heisenberg 1930, 988).

The physicist and philosopher Philipp Frank, a leading figure in the school of logical positivism, was pleased with what he saw as Dirac's philosophical position, namely that physical theory can only answer questions that relate to the outcome of experiments, whether real or imagined. It has nothing to say about reality as an abstract concept, separated from experiment or observation (Frank 1933, 63). Contrary to other reviewers, the mathematician Bernard Osgood Koopman found that *Principles* was more characterized by Dirac's profound intuition than any logical clarity in presentation. With respect to clarity and mathematical rigour, he preferred the writings of John von Neumann. Also contrary to most other reviewers, Koopman commented critically on the pedagogical quality of Dirac's book: "We feel that the usefulness of the book would have been enhanced by supplying it with an appendix, and by giving more references" (Koopman 1931, 495–496).[11] And he objected to Dirac's use of the hybrid terms "eigenvalue" and "eigenfunction" which should, he thought, preferably have been replaced by the English names "characteristic number" and "characteristic function." At the time the German words "Eigenwert" and "Eigenfunktion" were sometimes transcribed as "proper value" and "proper function," but Dirac decided to stick to the hybrid forms which he had used in his earlier publications.[12]

John Lennard-Jones, who at the time was professor of theoretical chemistry at Cambridge, agreed that *Principles*, for all it qualities, was not a masterpiece of pedagogy. "It would be idle to pretend that the book is easy to read," he said in an understatement. Although he found the book to be much too difficult for the uninitiated, he concluded that "it should be read by everyone who desires to keep in touch with modern physics" (Lennard-Jones 1931, 505–506).

Finally, it is worth mentioning that Einstein was impressed by Dirac's book, which he considered a most logically clear presentation of quantum mechanics. In a volume commemorating the centenary of the birth of Maxwell, Einstein reflected on the view of physical reality as expressed by the standard probabilistic interpretation of quantum mechanics. He said:

> Dirac, to whom, in my opinion, we owe the most logically perfect presentation of this theory, rightly points out that it appears, for example, to be by no means easy to give a theoretical description of a photon that shall contain within it the reasons that determine whether or not the photon will pass a polarizer set obliquely in its path. (Einstein 1931, 73)

This was a direct reference to the introductory chapter of *Principles*, in which Dirac had discussed in detail the polarization of photons. Although Einstein did not agree with Dirac's view of quantum mechanics, he appreciated the clarity and profoundness of his exposition.

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[11] Koopman, a student of George D. Birkhoff, did work on ergodic theory, dynamical systems, mathematical physics, and later operations research. Yet another review, by the Italian physicist Franco Rasetti, appeared in *Scientia* 51 (1932, 371). See also the reviews quoted below.

[12] The first English textbook in quantum mechanics, George Birtwistle's *The New Quantum Mechanics* (1928) used "eigenfunction" and "eigenwert."

10.5 Structure and Content

Compared with other textbooks in theoretical physics, the format of the first edition of *Principles* was unusual. With no illustrations and no index, it was not a reader-friendly work. Again in contrast to other books on quantum mechanics, it was completely ahistorical and contained almost no references to the research literature. To be precise, altogether it included twelve references in its 264 pages. Dirac admitted in his preface that his chosen way of representation had "necessitated a complete break from the historical line of development," but this he considered to be an advantage rather than a disadvantage. Although a considerable part of *Principles* was based on Dirac's own works and discoveries, there was no indication at all of which parts were his own contributions, nor were there any references to them. While some scientists use the medium of the textbook to communicate and advertise their own work, this was not the case with Dirac. Readers unacquainted with the development of quantum physics would not guess that most of the sections on transformation theory, the δ-function, radiation theory, and relativistic quantum mechanics were, in fact, about and based on the author's own works. This kind of anonymity does not imply that *Principles* was a neutral presentation of an accepted theory. Dirac clearly had an agenda in writing the book, namely to disseminate what he thought were the basic principles and proper methods of quantum mechanics. He wanted to *shape* a theory which had not yet found its final shape.

The book was basically divided in two parts of about equal size. The first part dealt with the principles and general formalism, followed by applications of the theory, including perturbation theory, collision problems, quantum statistics, and radiation theory. It ended with a chapter on the new relativistic theory of the electron. Dirac's original exposition is illustrated by Planck's constant, which in all other textbooks is introduced early on. But in *Principles* it only appeared on p. 95, in connection with the general commutation relations, as "a new universal constant having the dimensions of action." Only some lines later was it revealed that "[i]n order that the theory may agree with experiment," the new constant had to be the same as the one introduced by Planck. Incidentally, this was where Dirac introduced the symbol $\hbar$ ("Dirac's h") as shorthand for $h/2\pi$. Similarly, the Schrödinger wave equation appeared only on p. 104.

In the preface to the edition of 1930, Dirac stressed the abstract and unvisualizable nature of quantum mechanics and how different it was from classical physics. The aim of physics in the classical tradition was "to make assumptions about the mechanism and forces connecting […] observable objects, to account for their behavior in the simplest possible way." But the new developments, not only in quantum mechanics but also in relativity theory, had demonstrated that "nature works on a different plane." Nature's fundamental laws, Dirac said, "do not govern the world as it appears in our mental picture in any very direct way, but instead they control a substratum of which we cannot form a mental picture without introducing irrelevancies." Contrary to other works on quantum theory, which were based on the method of either matrix mechanics or wave mechanics, Dirac chose a more general representation. This representation, which he called the symbolic method, was harder to learn but "seems to go more deeply into the nature of things."

The first chapter, on "The Principle of Superposition," was purely qualitative, involving no mathematics. He carefully discussed the meaning of superposition by illustrating it with the case of polarization of light, emphasizing that "the superposition that occurs in quantum mechanics is of an essentially different nature from that occurring in the classical theory"

(Dirac 1930, I, 11).[13] Another very important term was the concept of "state," which he defined as referring to the condition of a system being independent of time, that is, to a region of four-dimensional space-time and not to three-dimensional space. "A system, when once prepared in a given state, remains in that state as long as it remains undisturbed," he wrote (I, 9). He further stated that if an observation is made on a system in any given state, "the result will not in general be determinate, i.e., if the experiment is repeated several times under identical conditions several different results may be obtained" (I, 10).

In the second edition of 1935, Dirac used the term "state" in a different sense, namely, to denote the condition of a physical system at a given time and not for all time. That is, he used it in a three-dimensional, non-relativistic sense, which might seem to be a retrograde step compared with the definition given in the first edition. However, Dirac motivated the change by arguing that it made the exposition clearer and also that "the fundamental ideas of the present quantum mechanics are in need of serious alterations at just this point" (II, v). He undoubtedly had in mind the problems of formulating a consistent relativistic theory of quantum electrodynamics with which he and other physicists were struggling at the time. Dirac perceived these problems to be so serious that he was willing to sacrifice the relativistic theory and perhaps even such a fundamental principle as the conservation of energy. As he wrote in a paper of early 1936:

> The present quantum mechanics [...] forms a satisfactory theory only when applied non-relativistically [...] and loses most of its generality and beauty when one attempts to make it relativistic. (Dirac 1936, 298; Kragh 1990, 168–173)

The second, somewhat enlarged edition of *Principles* retained the basic structure of the first edition, but was written in a less abstract and symbolic form. "This should make the work suitable for a wider circle of readers," Dirac said, adding that "the reader who likes abstractness for its own sake may prefer the style of the first edition" (II, v). Upon receiving a copy of the new edition, Heisenberg expressed his satisfaction with the work being "more human [*menschlicher*] than earlier,"[14] a response shared by many other physicists. For example, the American physicist Paul Epstein judged that the original version made "difficult reading, overtaxing the powers of abstraction of the less experienced student and making the book unsuitable as a classroom text." He was pleased with the changes made in the new edition, which made "the book clear and simple in all its parts, and there is no longer any reason why it should not prove of excellent service as a text in advanced courses" (Epstein 1935, 640–641).

Yet, not all reviewers were impressed by the pedagogical quality of the new, more *menschlich* edition. Charles Galton Darwin, at the time professor of physics at the University of Edinburgh, did not consider it more suitable as a textbook than the first edition. He complained that it was only helpful to those already familiar with quantum mechanics and that it lacked concrete examples to illustrate the general and formal exposition of the theory of quanta (Darwin 1935, 411–412).[15] Darwin had, since 1926, been a firm supporter of

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[13] References to the first edition are denoted by "I", those to the second edition by "II".

[14] Heisenberg to Dirac, 27 March 1935, quoted in (Brown 2006, 388).

[15] Koopman agreed that the second edition was not more suitable as a textbook on quantum theory than the first one: B. O. Koopman, *Bulletin of the American Mathematical Society* 42 (1936, 472–474). Given the completely ahistorical nature of *Principles*, it is remarkable that the second edition received an extensive review in *Isis*, the journal of the History of Science Society. See H. T. Davis, *Isis* 25 (1936, 493–496).

wave mechanics, which he saw as the only picture of the quantum world that yielded a visual representation and therefore understanding. Contrary to Dirac, he thought quantum theory should and could give insight into the reality of the subatomic realm; it was more than just a mathematical formalism to handle experimental data (Navarro 2009).

At the time Dirac completed *Principles*, in early 1930, he was much occupied with the "± difficulty" that arose from his relativistic wave equation, namely, how to interpret the negative-energy solutions in physical terms. In the final chapter of *Principles*, Dirac presented his new theory of the electron much as he had presented it in 1928. In dealing with the states formally referring to negative energies, he proposed that the antielectrons—unoccupied holes in the sea of negative energy states—were protons. The unifying idea of identifying antielectrons with protons, and thus reducing all matter to one elementary particle, appealed greatly to him. But of course he realized that it was hard "to account for the very considerable observed differences between electrons and protons, in particular their different masses." In the very last sentence of the book he stated optimistically: "Possibly the solution of this difficulty will be found in a better understanding of the nature of interaction" (I, 257).

This did not happen. In 1931 Dirac famously predicted the existence of positive electrons, which were subsequently discovered in cosmic rays and known as positrons. Much of the discussion in the second edition was identical with the one in the first, except that "proton" was now replaced by "positron." Concerning the negative-energy solutions, Dirac wrote that they referred to "a new kind of particle having the mass of an electron and opposite charge. Such particles have been observed experimentally and are called *positrons*" (II, 271).

Contrary to the presentation of quantum mechanics by Heisenberg, Born, Jordan and most other authors, in Dirac's presentation the analogy with classical mechanics played an important role. Although quantum mechanics differed radically from the laws and concepts of classical physics, on the formal level there was a great deal of similarity. "Practically all the features of the classical theory to which it owes its attractiveness can be taken over unchanged into the new theory," he wrote (I, 1). Dirac had originally arrived at his formulation of quantum mechanics by noticing a close analogy between the Poisson brackets of classical dynamics and the non-commuting products found by Heisenberg, and he continued to find the analogy significant. As he showed in *Principles*, by means of the Poisson formulation of the classical equations of motion, "one can in this way obtain a quantum theory of individual dynamical systems analogous to the classical theory" (I, 93). The emphasis on the classical analogy was a special feature of Dirac's textbook and reflected his own discovery of quantum mechanics in the years 1925–26.

Among the things *not* included in either of the editions was the complementarity principle, which played such an important role in Heisenberg's contemporary *Physikalische Prinzipien der Quantentheorie*. It was the purpose of Heisenberg's book to disseminate what he called the "Kopenhagener Geist der Quantentheorie," and the complementarity principle was a crucial part of this spirit. Whereas Dirac's purpose was to establish quantum mechanics on a logically satisfying basis suitable for calculations. Questions of interpretation were of secondary importance and mostly appeared implicitly. Although Dirac was, of course, familiar with Bohr's principle of complementarity, it was foreign to his way of thinking. He could express the substance of quantum theory without the airy castle of complementarity and consequently saw no reason to include it in his book. "I don't altogether like it," he said much later about complementarity. "It is rather indefinite [and] doesn't provide you with any equations which you didn't have before."[16] He thought that it might be useful for students preparing for examinations, but not for physicists doing research.

10.6 Dirac's Style of Physics

The symbolic method which was such a characteristic feature of *Principles*, the first edition in particular, was a main reason why many readers found the book difficult to understand. The method was based on "certain symbols which we say denote physical things [… and which] we shall use in algebraic analysis in accordance with certain axioms" (I, 18). Dirac wanted to present the general theory of quantum mechanics in a way that was as free as possible from physical interpretation:

> One does not anywhere specify the exact nature of the symbols employed, nor is such specification at all necessary. They are used all the time in an abstract way, the algebraic axioms that they satisfy and the connexion between equations involving them and physical conditions being all that is required. The axioms, together with their connexions, contain a number of physical laws, which cannot conveniently be analyzed or even stated in any other way. (ibid.)

Dirac's philosophy of physics, in the form that implicitly permeated much of his book, was markedly instrumentalist and abstract. Quantum physics was presented as a formal scheme that allowed the calculation of experimental results, and there was nothing more to it. In his lecture notes from 1927–28, he emphasized that the new theory "deals essentially only with observable quantities, a very satisfactory feature." Moreover: "[t]he theory enables one to calculate only observable quantities […] and any theories which try to give a more detailed description of the phenomena are useless."[17] The same message was spelled out in *Principles*, in both the first and the second edition. For example: "[t]he description which quantum mechanics allows us to give is merely a manner of speaking which is of value in helping us to deduce and to remember the results of experiments and which never leads to wrong conclusions" (I, 5). He added, significantly: "[o]ne should not try to give too much meaning to it." In his review of the book, Lennard-Jones focused critically on Dirac's instrumentalist attitude which he paraphrased as follows:

> A mathematical machine is set up, and without asserting or believing that it is the same as Nature's machine, we put in data at one end and take out the results at the other. As long as these results tally with those of Nature, […] we regard the machine as a satisfying theory. But so soon as a result is discovered not reproduced by the machine, we proceed to modify the machine until it produces the new result as well. (Lennard-Jones 1931, 505–506)

[16]AHQP, interview by Thomas S. Kuhn of 14 May 1963. On Dirac and complementarity, see (Heilbron 1985; Kragh 1990, 81–84). *Principles* was not alone in ignoring complementarity: Among 43 textbooks on quantum mechanics published between 1928 and 1937, 40 included a treatment of the uncertainty principle, but only eight of them mentioned the complementarity principle.
[17]Notes for "Lectures on Modern Quantum Mechanics" (AHQP).

This was Dirac's view of quantum theory, but it was not shared by Lennard-Jones who wanted a "rather more ambitious" object for theoretical physics. He deplored that the quantum theorist, at least according to Dirac, "must for ever abandon any hope of providing a satisfying description of the whole course of phenomena." In this respect, Lennard-Jones agreed with Darwin.

Dirac, never much of a philosopher, was in general agreement with the Bohr-Heisenberg view of quantum theory, including the interpretation of the measurement process and the nature of the principle of indeterminacy. Although his book did not refer explicitly to philosophical issues, it did much to disseminate certain views of the Copenhagen school to a generation of young physicists. In accordance with Bohr, in the preface to the first edition, Dirac called attention to "the increasing recognition of the part played by the observer himself in introducing the regularities that appear in his observations." This he considered "very satisfactory from a philosophical point of view" (I, v). Also with regard to determinism and causality, he shared the view of Bohr and his circle of physicists. Quantum mechanics was fundamentally a probabilistic theory, and "the most that can be predicted is the probability of occurrence of each of the possible results" (I, 4). The uncertainty in the initial conditions of a physical system implied indeterminism and a failure of causality, which "from this point of view [is] due to a theoretically necessary clumsiness in the means of observation" (ibid.). But Dirac also pointed out that probability densities and currents, as given by the Schrödinger equation, evolve classically: "[t]he differential equations that express the causality of classical mechanics do not get lost, but are all retained in symbolic form, and indeterminacy appears only in the application of these equations to the results of observation" (II, 4).

Although one can reasonably label Dirac, at the time he wrote *Principles*, a quantum instrumentalist, there are more grounds to doubt that he shared the positivistic view of physics that characterized Heisenberg, Bohr, Jordan, and some other advocates of the Copenhagen school. As Born remarked in 1936, "Whereas he [Dirac] declares himself content with the formulae and uninterested in the question of an objective world, positivism declares the question to be meaningless" (Born 1936, 13). On the other hand, Dirac's disagreements with the Bohr-Heisenberg view were relatively minor. It is true that he came to side with Einstein, at least to some extent, and to criticize the Copenhagen-Göttingen camp, but this was only much later in life.[18]

Regarding Dirac's later advocacy of mathematical beauty as a royal road to progress in fundamental physics, it is noteworthy that, in the early 1930s, he still considered mathematics more from the perspective of an engineer than a mathematician. Although quantum mechanics, as presented in *Principles*, was said to be "essentially mathematical," this referred only to the formalism. "All the same," Dirac wrote, "the mathematics is only a tool and one should learn to hold the physical ideas in one's mind without reference to the mathematical form" (I, vi). Indeed, while many physicists and students found Dirac's book heavily mathematical, mathematicians were unimpressed by the way he used mathematics. "Dirac permits himself a number of mathematical liberties," wrote the mathematician Garrett Birkhoff in a letter.[19] One of those liberties was the δ-function, which Dirac had introduced in 1927, and

[18]Bokulich (2008) argues that Dirac sided with Einstein in the debate over the interpretation of quantum mechanics, and that he did so even in the early phase of the debate. However, her evidence is unconvincing as it is limited to some of Dirac's writings from the 1960s and 1970s.

[19]Birkhoff to Edwin Kemble, 3 March 1933, quoted in (Kragh 1990, 279).

which he dealt with in § 22 of *Principles*. For him it was merely "a convenient notation" that might be used freely for dealing with the representatives of the abstract symbols, as though it were a continuous function, without leading to incorrect results" (I, 64). Mathematicians looked upon the δ-function in a very different way and did not appreciate Dirac's more intuitive use of mathematics, see (Peters 2004).

The dissatisfaction of contemporary mathematicians with Dirac's methods was expressed by John von Neumann, who in his *Mathematische Grundlagen der Quantenmechanik* of 1932 undertook to provide quantum mechanics with a proper mathematical foundation. In the preface of the book, he said about the method of Dirac, as presented in *Principles*, that it "in no way satisfies the requirements of mathematical rigor—not even if these are reduced in a natural and proper fashion to the extent common elsewhere in theoretical physics" (von Neumann 1943, 2). In a comment in the *Mathematical Gazette*, the American physicist Henry Margenau contrasted Dirac's use of mathematics to that of von Neumann:

> While Dirac presents his reasoning with admirable simplicity and allows himself to be guided at every step by physical intuition—refusing at several places to be burdened by the impediment of mathematical rigor—von Neumann goes at his problems equipped with the nicest of modern mathematical tools and analyses it to the satisfaction of those whose demands for logical completeness are most exacting.[20]

Contrary to Weyl's textbook of 1928, which was based on the mathematical theory of groups, group theory was absent from *Principles* both in its first and later editions. Dirac was familiar with the new, mathematically abstract way of representing quantum theory, but he did not find it either more fundamental or very helpful. He preferred to treat group theory as part of quantum mechanics, which for him was the general science of non-commuting quantities (Dirac 1929).

10.7 Concluding Remarks

As I have indicated, *Principles* was a difficult work and not pedagogical in the ordinary sense. Dirac based it to a large extent on his lectures of 1927–29 and, after having completed it, used it for the lectures on quantum mechanics he gave over most of the next four decades. During the 1930s, there was another regular lecture course on quantum physics in Cambridge, given by Alan Wilson in the fall (Michaelmas) term, while Dirac gave his lectures in the spring (Lent) term. Wilson's course was more practically oriented, based on applications of the Schrödinger equation (Wilson 1984).

Although Dirac did not specifically refer to his book as a textbook, in the preface to the first edition he did mention students, and he seems to have regarded it as both a textbook and an exposition of the principles of quantum theory aimed at physicists. I doubt if he gave much thought to the intended readership. Because of Dirac's lectures, which closely followed his book, *Principles* exerted considerable influence on a generation of Cambridge physicists. "His influence was not very great as a teacher," Mott recalled, except that "he always, of course, has given this lecture on his book with admirable character."[21]

[20] Quoted in (Jammer 1966, 367). On Dirac's pragmatic use of mathematics, see (Bueno 2005).
[21] Interview with Mott in 1963 (AHQP). Quoted in (Kragh 1990, 253).

I do not know how much and at which levels *Principles* was used as a textbook outside Cambridge, but my guess is that it was not widely used for lectures or in the classroom. Even if this guess were right, however, it was much used by physicists, both young and more experienced. The number of copies sold speaks for itself. It rarely happens that textbooks are cited in research papers, but *Principles* was an exception to the rule. In the physics papers of the 1930s, there were many references to Dirac's book, which probably exerted a greater influence on research physicists than students.

In the early stages of a new science, discipline, or research field, textbooks play an important role by legitimating the field and formulating the principles on which it builds. Whether explicitly or implicitly, the first generation of textbooks articulate the constitutive features of the new research field, which is particularly important in changes of a more revolutionary nature, such as quantum mechanics. Because the field is not yet fully consolidated, early textbooks may differ considerably in their understanding of the field, both as to content and methodology. It is almost inevitable that what an author presents has the character of a partisan text, at least in the sense that the book reflects the author's view of the new field of science.[22] Dirac's *Principles of Quantum Mechanics* was far from polemical, but it was nonetheless a textbook that conveyed a view of quantum mechanics that may well be called partisan or even personal.

Abbreviations and Archives

AHQP	Archive for History of Quantum Physics. American Philosophical Society, Philadelphia

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[22] For the case of early textbooks in quantum chemistry, see (Gavroglu and Simões 2000).

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