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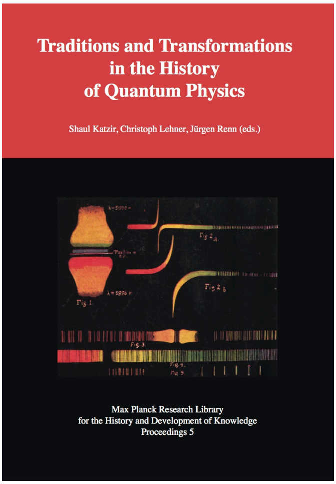

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Chapter 10
Tsung-Sui Chang's Contribution to the Quantization of Constrained Hamiltonian Systems

Xiaodong Yin, Zhongyuan Zhu, and Donald C. Salisbury

The quantization of constrained systems is one of the cornerstones of modern elementary dynamical theories. Important fundamental physical theories, such as quantum electrodynamics, quantum chromodynamics, electro-weak unified theory and string theories make use of it. The most widely used, currently canonical formulation for the quantization of constrained Hamiltonian systems was proposed by Paul Dirac in 1950 (Dirac 1950; 1951; 1964), and independently by Peter Bergmann and his collaborators (Bergmann and Brunings 1949; Bergmann, Penfield, et al. 1950; Anderson and Bergmann 1951). Later, in 1967, Ludvig Fadeev and Victor Popov made important progress in the path-integral quantization of the Yang-Mills field (Fadeev and Popov 1967).

In this paper, we emphasize the contribution of a Chinese theoretical physicist, Professor Tsung-Sui Chang, to this topic. In 1946, Chang pointed out that the previous canonical formulations of constrained systems could not be applied to quantum theory because they did not provide a method for dealing with one of the key features of the analogous classical theories—the appearance of undetermined multipliers. Chang worked out a feasible quantization procedure for such systems.

In the following section, we present a summary of Chang's education, training, and professional development from the 1930s to his death in 1969. In subsequent sections, we outline theoretical developments in the field leading up to Chang's own advances.

10.1 Biographical Overview

Chang was born in Hangzhou, Zhejiang Province, on 12 July 1915. He studied physics at Yenching University in 1930, then in 1931 he joined the Physics Department of Tsinghua University, headed by Wu Youxun. It was one of the most prestigious universities in China. In 1934, Chang began a masters degree program under Wu's supervision. Wu recommended that he continue his training at Cambridge University.

In August of 1936, Chang entered the Mathematics Department of Cambridge University, supported by the Boxer Indemnity. He studied statistical physics as a doctoral student under Ralph Howard Fowler. Chang completed several important works on cooperative phenomena (solid solution, adsorption). The well-known text "Statistical Thermodynamics" by Fowler and Edward Guggenheim (1939) includes a section "The Combinatory Formulae of Chang."

After receiving his doctorate at Cambridge in 1938, Chang decided to extend his research field beyond statistical physics to quantum field theory. In 1938, Fowler endorsed Chang's application to Bohr:[1]

> I think I can whole-heartedly recommend him to you. He has done very well in his two years in Cambridge showing very considerable initiative and skill in developing the formal consequences such as order and disorder in alloys. I think you would find him very pleasant to deal with, and thoroughly industrious and able.

In 1939, Chang went to the Theoretical Physics Institute at the University of Copenhagen as a postdoctoral fellow and commenced his research on quantum field theory. His academic career began in earnest with stays at different locations in Europe, including: Copenhagen (September 1938 to February 1939), Zurich (February to June 1939), and Paris (June to October 1939). His acquaintances included Wolfgang Pauli, Niels Bohr and Aage Bohr. In Copenhagen, Chang lived in Bohr's home and established a very good relationship with Bohr's family. He completed two articles, "The Azimuthal Dependence of Processes Involving Mesons" (Chang 1940) was published in 1940, then another article on the nature of pseudo-scalar mesons. The latter publication was delayed until 1942 due to Chang's 1939 return to China and wartime communication difficulties.

Once back in China, Chang became the youngest professor of physics at the National Central University in Chongqing. For the next six years, during the Sino-Japanese War, Chang continued his research on statistical physics and quantum field theory, including the quantization of constrained systems. He published about ten articles during the difficult war period. Meanwhile, he was eager to pursue an international academic exchange. Chang's hope was fulfilled at the end of 1945 when he had the opportunity to return to Cambridge, thanks to Joseph Needham, who was the head of the Sino-British Science Cooperation Bureau in China, and Dirac. The Bureau provided information and aid for institutions and universities in China in wartime by sending papers to Western journals, offering scientific instruments and sending Chinese scholars to the United Kingdom.

[1]Correspondence by Fowler to Bohr on 8 June 1938, Niels Bohr Archive.

During 1944–1946, the Sino-British Science Cooperation Bureau assisted eight Chinese professors in going abroad, including Chang.

In addition to his connection to Bohr, Chang had a close relationship with Dirac, one of Fowler's previous students. Dirac had already become a well-known professor during Chang's first visit to Cambridge. Dirac's research style served as a model for Chang. It was under Dirac's influence that Chang undertook the study of quantum field theory in 1939. Further, Dirac recommended that Chang teach a Cambridge course on quantum field theory,[2] and they had many discussions on this and other topics.

Chang's first two papers on constrained systems published in Britain were communicated by Dirac, and in the third paper, published in 1947, he thanked Dirac for his interest and discussions.

In the autumn of 1947, Dirac went to the Princeton Institute for Advanced Studies for a short-term visit. He suggested that Chang join him there. Chang spent six months with Dirac in New Jersey. He was then invited to work at the Carnegie Institute of Technology in Pittsburgh, Pennsylvania, for five months.

In the autumn of 1949, Chang left the United States and returned again to China. He successively became a professor in the Physics Department of Peking University, Beijing Normal University and the Institute of Mathematics of the Chinese Academy of Sciences. He also became an academician of the Chinese Academy of Sciences in 1957. He is recognized as one of the founders of quantum field research in China. Despite his successes, he suffered during the Cultural Revolution period and committed suicide on 30 June 1969 at 54 years of age.

10.2 Studies on Constrained Hamiltonian Systems before Chang's Work

10.2.1 Initial Studies

To better understand Chang's contributions, we now turn to initial studies of constrained systems. The challenge of quantizing a classical constrained dynamical model was present at the birth of quantum field theory. Classical electromagnetism is such a theory, so efforts to describe the quantum interaction of the field with charged particles needed to address the problem of constraints directly. Historically, the first such successful theory, with full quantum electromagnetic interaction, was introduced by Dirac in 1927 (Dirac 1927). With regard to the electromagnetic field itself, Dirac followed a method advanced by Pascual Jordan in 1926 in a joint publication with Born and Heisenberg (Born, Jordan, and

[2]Cambridge University Reporter, Reporter issues for the academic year 1936–1937, Vol. 67, 682; 1937–1938, Vol. 68, 633.

Figure 10.1: Tsung-Sui Chang, 1915–1969, Source: Prof. Yi Ci Chang.

Heisenberg 1926). The basic method is to decompose the radiation field into harmonic oscillators, so the quantization of the electromagnetic field was reduced to the quantization of these oscillators. The Dirac scheme dealt exclusively with transverse components of the field and was therefore not obviously relativistically covariant. Subsequently, in 1929, Heisenberg and Pauli established a canonical quantization procedure for general quantum fields (Heisenberg and Pauli 1929; 1930). However, when applying their method to the electromagnetic field, they encountered a stubborn difficulty that was eventually overcome by Heisenberg: the classical momentum conjugate to the scalar potential of electromagnetic fields vanishes identically. This means that the related canonical degrees of freedom were not independent. The canonical variables were subject to constraints. The immediate consequence was the contradictory conclusion that the commutator of this vanishing momentum with the scalar potential would not vanish. Thus, a procedure was needed to avoid this contradiction.

10.2.2 Earlier Approaches to Constrained Systems by Rosenfeld and Dirac

Obvious problems existed in early approaches to the quantization of the electromagnetic field.[3] The first Heisenberg-Pauli method added a new term to the

[3] See (Salisbury 2009a) for a detailed analysis.

Lagrange function, multiplied by a small parameter ϵ. This had the effect that the momentum no longer vanished. However, the ϵ term destroyed the manifest gauge invariance of the Lagrangian. The second Heisenberg-Pauli method set the scalar potential A_0 equal to zero, thus destroying manifest Lorentz covariance. Gauss's law was then imposed as an initial condition on quantum states. In this same paper, they showed that a method that had—in the meantime—been put forth by Fermi was equivalent to adding a Lorenz gauge-fixing term to the Lagrangian, and this gauge condition also needed to be imposed as an initial quantum condition (Fermi 1929; Heisenberg and Pauli 1930).

Pauli invited the young Rosenfeld to join him in Zurich in 1929 to establish a firmer theoretical foundation for the methods that he, Heisenberg and Fermi had employed in their treatment of quantum electromagnetic field theory. Rosenfeld set himself the task of formulating a Hamiltonian procedure for dealing with local gauge symmetries in the two fundamental interactions that were known at the time, electromagnetism and Einstein's curved space-time gravitation. From the start he focused on the problem of implementing the full gauge symmetry group as a canonical transformation group acting on the phase space field variables. He made enormous progress in this effort, although his results were largely unknown (or in the case of Dirac, perhaps forgotten) by subsequent researchers. There is no indication that Chang was acquainted with Rosenfeld's work, although we do know that Dirac was aware of it in 1932 (Salisbury 2009a).

Rosenfeld showed that, as a consequence of local Lagrangian symmetries, identities arise that relate the canonical momenta and configuration variables (when the former are understood as functions of the configuration variables and their time derivatives). Following Rosenfeld, we represent the ensuing constraining relations as $F^r(p,q) = 0$, where the index r ranges over the total number of such so-called primary constraints. It followed that the time development for given initial conditions was not unique. Furthermore, Rosenfeld showed that this time development could be represented in first-order Hamiltonian form, with a Hamiltonian

$$H = H_0(p,q) + \lambda^r F_r(p,q), \tag{10.1}$$

where the λ^r are arbitrary space-time functions.[4] The Hamilton equations are then

$$\frac{dq^\alpha}{dt} = \frac{\partial H_0}{\partial p_\alpha} + \lambda^r \frac{\partial F_r}{\partial p_\alpha} \tag{10.2}$$

[4]We assume here for the sake of simplicity that the number of physical variables q is finite. The extension to field theory is straightforward.

and

$$\frac{dp_\alpha}{dt} = -\frac{\partial H_0}{\partial q^\alpha} - \lambda^r \frac{\partial F_r}{\partial q^\alpha}. \tag{10.3}$$

Even though Rosenfeld presented an explicitly q-number version of his formalism in his 1930 article, he did not address the question of how one would or could incorporate the arbitrary functions λ^r into the quantum theory. However, as pointed out elsewhere (Salisbury 2009a), he had all the tools required to construct gauge invariant objects using his symmetry group generators. In a review of quantum electromagnetism published two years later (Rosenfeld 1932), he simply reverted to the Fermi scheme, after having convinced himself that his prior analysis justified the procedure. Curiously, however, he did not express this conviction in writing.

In 1933, Dirac published a paper entitled "Homogenous Variables in Classical Dynamics" in which he considered a far narrower class of models than Rosenfeld had examined, not citing him even though it is clear from an exchange of letters in 1932 that Dirac was familiar with Rosenfeld's work. Dirac wrote in this paper:

> The well-known methods of classical mechanics, based on the use of a Lagrangian or Hamiltonian function, are adequate for the treatment of nearly all dynamical systems met with in practice. There are, however, a few exceptional cases to which the ordinary methods are not immediately applicable. For example, the ordinary Hamiltonian method cannot be used when the momenta p_r, defined in terms of the Lagrangian function L by the usual formulae $p_r = \partial L/\partial \dot{q}_r$, are not independent functions of the velocities. (Dirac 1933)

Dirac used the electromagnetic field and the massless relativistic particle as examples to illustrate his program. The outcome is the same Hamilton equations exhibited by Rosenfeld, with the same arbitrary functions (though designated by Dirac by ρ rather than λ). Referring to Dirac's paper, in 1946 Chang observed that the appearance of these arbitrary functions seemed to preclude passage to a quantum theory.

10.3 Chang's Contributions to Hamiltonian Systems

From July 1944 to June 1946, Chang published three papers on the quantization of constrained systems: "A Note on the Hamiltonian Theory of Quantization" (Chang 1945), "A Note on the Hamiltonian Equations of Motion" (Chang 1946), and "A Note on the Hamiltonian Theory of Quantization (II)" (Chang 1947).

The first and second papers were completed under very difficult conditions in Chongqing, a southwestern city in China, during the Sino-Japanese War. They were published in the Proceedings of the Royal Society of London and Proceedings of the Cambridge Philosophical Society, respectively, and were communicated by Dirac. In the second and third papers, Chang expressed his thanks to Dirac for discussions. The third paper, completed at Cambridge University, was the most extensive, summarizing some results from the previous two, and is the principal subject of our analysis. This 1947 paper is divided into three sections in which Chang discusses the need for dealing with the arbitrary functions λ, a proposal for quantizing models in which constraints are imposed at the Lagrangian level through the use of Lagrange multipliers, and a proposal for quantizing systems with primary constraints.

First, Chang observed, in referring to Dirac's 1933 paper,

> The Lagrangian equations for cases with missing momenta have been studied some time ago by Dirac by making use of homogeneous variables. It was shown that the equations of motion can always be put in canonical form. However, the final equations still contain quantities of the nature of unknown Lagrange multipliers, and are thus not suitable for passing to a quantum theory. (Chang 1947)

This may be the first published observation of the challenge that the undetermined functions posed for the canonical quantization program.

10.3.1 Models with Lagrange Multipliers

The first models that Chang considered were models in which constraints were imposed "by hand" through the use of Lagrange multipliers. He considered systems for which the Lagrangian contained only first derivatives $q^\alpha_{,\mu} := \frac{\partial q^\alpha}{\partial x^\mu}$. He supposed that variations of the action

$$I := \int L\left(q^\alpha, q^\alpha_{,\mu}, x\right) d^4x, \tag{10.4}$$

were subject to f auxiliary conditions

$$f^{(\xi)}(q^\alpha, q^\alpha_{,\mu}, x) = 0 \ \ (\xi = 1, 2, \dots, f). \tag{10.5}$$

Then it followed that

$$\frac{\partial L}{\partial q^\alpha} + \sum \mu_{(\xi)} \frac{\partial f^{(\xi)}}{\partial q^\alpha} - \frac{\partial}{\partial x^\mu}\left(\frac{\partial L}{\partial q^\alpha_{,\mu}} + \sum \mu_{(\xi)} \frac{\partial f^{(\xi)}}{\partial q^\alpha_{,\mu}}\right) = 0. \tag{10.6}$$

Eqs. (10.5) and (10.6) are the field equations. The Lagrange multipliers μ_ξ are understood to depend on the space-time coordinates, represented collectively by the symbol x, where we will take them to be real, with $x^\mu = \{ct, \vec{x}\}$.[5] Superscripts $\mu, \nu, \ldots$ run from 1 to 4, while superscripts $r, s, \ldots$ go from 1 to 3. The q^α are the dynamical field variables.

Chang claimed to have achieved a first-order canonical form for his field equations by first implicitly defining the functions $b^\alpha(p_\alpha, q^\alpha, q^\alpha_{,r}, x)$ and $\eta_\xi(p_\alpha, q^\alpha, q^\alpha_{,r}, x)$ through the relations

$$f^{(\xi)}(q^\alpha, q^\alpha_{,r}, b^\alpha, x) = 0 \tag{10.7}$$

and

$$p_\alpha - \frac{\partial}{\partial b^\alpha} L(q^\alpha, q^\alpha_{.r}, b^\alpha, x) - \eta_\xi \frac{\partial}{\partial b^\alpha} f^{(\xi)}(q^\alpha, q^\alpha_{.r}, b^\alpha, x) = 0, \tag{10.8}$$

where a sum over ξ is understood. He then defined the Hamiltonian to be

$$H(p_\alpha, q^\alpha, q^\alpha_r, x) := -L(q^\alpha, q^\alpha_r, b^\alpha, x) + p_\alpha b^\alpha, \tag{10.9}$$

where a sum over α is understood.

Using this Hamiltonian function, the canonical equations became

$$\frac{\partial q^\alpha}{\partial t} = \frac{\delta \widetilde{H}}{\delta p_\alpha(\vec{x})} = \frac{\partial H}{\partial p_\alpha} \tag{10.10}$$

and

$$\frac{\partial p_\alpha}{\partial t} = -\frac{\delta \widetilde{H}}{\delta q^\alpha(\vec{x})} = -\left(\frac{\partial H}{\partial q^\alpha} - \frac{\partial}{\partial x^r}\frac{\partial H}{\partial q^\alpha_{,r}}\right), \tag{10.11}$$

where $\widetilde{H} := \int H d^3x$.

Substituting for H we obtain

$$\frac{\partial q^\alpha}{\partial t} = b^\alpha + \eta_\xi \frac{\partial f^{(\xi)}}{\partial b^\beta}\frac{\partial b^\beta}{\partial p_\alpha} \tag{10.12}$$

[5] Chang used $x_4 := ict$.

and

$$
\begin{aligned}
\frac{\partial p_\alpha}{\partial t} &= \frac{dL(q^\alpha, b^\alpha, x)}{dq^\alpha} + \eta_\xi \frac{df^{(\xi)}(q^\alpha, b^\alpha, x)}{dq^\alpha} \\
&- \frac{\partial}{\partial x^r}\left\{\frac{dL(q^\alpha, b^\alpha, x)}{dq^\alpha_{,r}} + \eta_\xi \frac{df^{(\xi)}(q^\alpha, b^\alpha, x)}{dq^\alpha_{,r}}\right\},
\end{aligned}
\tag{10.13}
$$

where we define the total derivative with respect to q^α as

$$
\frac{dL(q^\alpha, b^\alpha, x)}{dq^\alpha} := \frac{\partial L(q^\alpha, b^\alpha, x)}{\partial q^\alpha} + \frac{\partial L(q^\alpha, b^\alpha, x)}{\partial b^\beta}\frac{\partial b^\beta}{\partial q^\alpha}. \tag{10.14}
$$

Chang claimed that eq. (10.12) yielded

$$
\frac{\partial q^\alpha}{\partial x_4} = b^\alpha,
$$

failing to mention that this form is achievable only by requiring that

$$
\eta_\xi \frac{\partial f^{(\xi)}}{\partial b^\beta}\frac{\partial b^\beta}{\partial p_\alpha} = 0.
$$

He also did not observe that additional restrictions regarding the q^α-dependence of the constraining relations (10.5) arise by requiring that the dynamical eq. (10.13) be equivalent to the Euler-Lagrange field eq. (10.6).

Generally this means that there are severe limitations to the applicability of Chang's method. We can, however, give a simple illustrative example in which the procedure may be implemented. We consider a system with

$$
L = \frac{1}{2}\left(\frac{dq}{dt}\right)^2
$$

and an auxiliary condition

$$
f = \frac{dq}{dt} = 0.
$$

Then, employing Chang's symbols, we have

$$
\begin{aligned}
f &= b = 0, \\
p &= \frac{\partial L}{\partial b} + \eta \frac{\partial f}{\partial b} = b + \eta = \eta, \\
H &= pb - \frac{1}{2} b^2 = 0, \\
\{q, p\}_{P.B.} &= 1, \\
\dot{q} &= \frac{\partial H}{\partial p} = 0, \\
\dot{p} &= -\frac{\partial H}{\partial q} = 0.
\end{aligned}
$$

We note also that the relevant additional consistency condition that is required to achieve the correct Euler-Lagrange equations is

$$
\eta \frac{df(q,b)}{dq} = \eta \left(\frac{\partial f}{\partial q} + \frac{\partial f}{\partial b} \frac{\partial b}{\partial q} \right) = 0. \tag{10.15}
$$

This does vanish since f is independent of q, and b vanishes identically. So although we have a non-trivial model with a Lagrange multiplier that leads to a self-consistent quantum theory here, it is dubious that the procedure could enjoy widespread applicability.

10.3.2 Models with Missing Momenta

Chang next discussed models which possessed missing momenta, i.e., models that in our current terminology have primary constraints. Chang identified those momenta as "missing" that vanished due to the absence of the time derivative of the conjugate configuration variable in the Lagrangian. He assumed that a transformation of these variables had been undertaken so that all primary constraints would take this simple form, thus dividing the configuration variables into a set q^α such that the $q^\alpha_{,0}$ could be written in terms of the conjugate momenta p_α, and another set Q^l with momenta $P_l = 0$. Chang appears to have been the first to recognize in print that this decomposition was achievable at the Lagrangian level, although he did not note that to implement it, it might be necessary to add total derivative terms to the Lagrangian. Dirac added such terms for general relativity in 1958 (Dirac 1958).

In terms of these variables, the Lagrangian field equations take the form

$$\frac{\partial L}{\partial q^\alpha} - \frac{\partial}{\partial x^\mu}\left(\frac{\partial L}{\partial q^\alpha_{,\mu}}\right) = \frac{\delta \tilde{L}}{\delta q^\alpha} - \frac{\partial p_\alpha}{\partial t} - = 0 \tag{10.16}$$

and

$$f_l(q^\alpha, Q^l, q^\alpha_{,0}, x) := \delta\tilde{L}/\delta Q^l = 0, \tag{10.17}$$

where $\tilde{L} := \int L d^3 x$. Thus Chang recognized that the relations (10.17) were themselves constraints, in addition to the primary constraints.[6]

He then presented a procedure about which he explicitly recognized that it was applicable only when the constraints could be employed to eliminate the Q^l as independent variables. That is, he assumed that the relations

$$p_\alpha - \frac{\partial L(q^\alpha, B^l, b^\alpha, x)}{\partial b^\alpha} = 0 \tag{10.18}$$

and

$$f_l(q^\alpha, B^l, b^\alpha, x) = 0 \tag{10.19}$$

could be solved for

$$Q^l =: B^l(q^\alpha, q^\alpha_{,r}, p_\alpha, x). \tag{10.20}$$

Then under these circumstances, the Hamiltonian

$$H(p_\alpha, q^\alpha, x) = -L(q^\alpha, B^l, b^\alpha, x) + p_\alpha b^\alpha$$

delivers

$$\frac{\partial q^\alpha}{\partial t} = \frac{\partial H}{\partial p_\alpha} = b^\alpha(q, q_{,r}, p, x) \tag{10.21}$$

and

$$\frac{\partial p_\alpha}{\partial t} = -\frac{\delta \tilde{H}}{\delta p_\alpha(\vec{x})} \tag{10.22}$$

and is equivalent to the original Lagrangian field equations.

Thus Chang recognized correctly that systems may exist where the constraints can be solved. And in this case, upon passing to the quantum theory, one can have canonical Hamiltonian equations and commutation relations just for the

[6] In the following section, Chang showed that in gauge covariant models these constraints arise due to the demand that primary constraints be preserved under time evolution.

independent variables p_α and q^α with the Q^l becoming functions of p_α, q^α. He did not observe that this circumstance arises when the constraints have non-vanishing Poisson brackets among themselves, or in the language that was introduced by Dirac in 1950, when the constraints are second class (Dirac 1950).

The models for which the constraints cannot be solved belong to another type. Chang also discussed a limited version of this case, which we address in the next subsection. This is the situation with gauge theories for which the constraints do have vanishing Poission brackets among themselves. Dirac called such constraints first class. So although Chang did not characterize them in this way, he was certainly aware that there existed two kinds of constrained systems, and he presented a preliminary procedure for quantizing them.

Dirac was one of the first to propose a quantization procedure that dealt with both types. In gauge theories, he noted that gauge conditions need to be invoked, and he introduced modified brackets that respected these conditions (Dirac 1950). It seems likely that Chang's work would have influenced Dirac's movement in this direction.

10.3.3 Gauge Covariant Models

Having realized that his program for second class constraints did not work for systems like electromagnetism, Chang then embarked on an alternate approach. He considered a system in which the Lagrangian L is such that some of the p_α are missing, but with the assumption that L is increased by an amount $F^L(x)$ under the transformation

$$q^\alpha(x) \to q^\alpha(x) + F^\alpha(x).$$

We represent the resulting variations with the symbol $\bar{\delta}$, writing $\bar{\delta} q^\alpha = F^\alpha$. The resulting variation of the Lagrangian is

$$\bar{\delta} L = \frac{\partial L}{\partial q^\alpha} F^\alpha + \frac{\partial L}{\partial q^\alpha_{,\mu}} F^\alpha_{,\mu} =: F^L. \tag{10.23}$$

Chang remarks that the variation he is considering "is not the same as an ordinary gauge transformation," but he calls it a gauge transformation "for lack of a better name." Yet he does assume that his Euler-Lagrange equations are covariant under this transformation. This result follows immediately from the assumption that F^L is a function only of x (and not of the dynamical variables). He is perhaps assuming, at least initially, that the F^α do not depend on arbitrary space-time functions. Also, general gauge transformations have a dependence on the dynamical variable, i.e., $\bar{\delta} q^\alpha = F^\alpha(q^\beta, q^\beta_{,\mu}, x)$. Such is the case, for example, with general relativity and also with the homogeneous models that were treated by Dirac (1933). In any case, the electromagnetic model that he cites specifically as susceptible to his analysis is clearly gauge covariant in the current sense.

Next, Chang points out that when the field equations are satisfied, one obtains, after performing a spatial integration by parts and letting the gauge variations vanish at spatial infinity,

$$\frac{\partial}{\partial t}\int d^3x\left(F^\alpha \frac{\partial L}{\partial q^\alpha_{,0}}\right) - \int d^3x F^L = 0. \tag{10.24}$$

At this point, Chang assumes that the F^α involve arbitary functions Φ of the time up to order u, and this leads him to a remarkable result: it follows from eq. (10.24) that the coefficients of each order of time derivative must separately vanish. This result was already known, to be sure, to Rosenfeld in 1930—but it was independently rediscovered in 1951 by James L. Anderson and Bergmann and is generally attributed to them (Anderson and Bergmann 1951). Indeed, these authors introduced the terminology that is still in use today. The requirement that primary constraints be preserved under time evolution may lead to secondary constraints. These in turn may lead to tertiary constraints, and so on.

Chang notes that the highest derivative term, $\frac{\partial^u \Phi}{\partial t^u}$, that appears in eq. (10.24) will not involve a time derivative of momenta. This term is

$$\int d^3x\left(\frac{\partial F^\alpha}{\partial t} p_\alpha\right), \tag{10.25}$$

and isolating the coefficient of $\frac{\partial^u \Phi}{\partial t^u}$ in the integrand, we deduce the existence of primary constraints that Chang represents as $g_u(q,p) = 0$. But then the coefficient of $\frac{\partial^{u-1} \Phi}{\partial t^{u-1}}$ will be in the form

$$g_{u-1}(p_{,0}, p) := \frac{\partial g_u(q,p)}{\partial t} - C_{u-1}(q,p) = 0. \tag{10.26}$$

In other words, the preservation of the primary constraint requires the existence of a secondary constraint $C_{u-1}(q,p) = 0$, and so on. Thus eq. (10.24) may be rewritten as

$$\int d^3x\left\{g_0(p,p_{,0})\Phi + g_1(p,p_{,0})\frac{\partial \Phi}{\partial t} + \dots + g_u(p,p_{,0})\frac{\partial^u \Phi}{\partial t^u}\right\} = 0 \tag{10.27}$$

with

$$g_0 = g_1 = \dots g_u = 0. \tag{10.28}$$

Actually, Chang did not state explicitly that constraints C would arise, but it is an immediate consequence of the relation (10.24).

Although Chang did not provide a concrete example, it is instructive to take the electromagnetic field as an example to illustrate his formulation—and identify one shortcoming. As mentioned above, Chang did note that the electromagnetic model satisfied his assumptions regarding the gauge invariance of the Lagrangian. This is true, provided that the charged source current is understood to be a non-dynamical external field, as we highlight below.

The Lagrangian for this model is

$$L = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + j^\mu A_\mu, \tag{10.29}$$

where

$$F_{\mu\nu} := A_{\nu,\mu} - A_{\mu,\nu} \tag{10.30}$$

and j^μ is an external current. Then the conjugate momenta are

$$p^a = \frac{\partial L}{\partial \dot{A}_a} = F^{a0} = -\dot{A}_a - A_{0,a}, \tag{10.31}$$

while $p^0 =: P$ vanishes identically.

The Lagrangian equations of motion are

$$\partial_0 F^{0a} + \partial_b F^{ba} + j^a = 0 \tag{10.32}$$

and

$$\partial_a F^{a0} + j^0 = 0. \tag{10.33}$$

The gauge variation is

$$\bar{\delta} A_\mu = \Phi_{,\mu}. \tag{10.34}$$

The variation of the Lagrangian under this transformation is

$$\bar{\delta} L = F^{\mu\nu} \Phi_{,\mu\nu} + j^\mu \Phi_{,\mu} = j^\mu \Phi_{,\mu} =: F^L. \tag{10.35}$$

Thus, provided that j^μ is a prescribed function of x, the variation may be written as a total time derivative, and the variation is a true gauge transformation.

Performing integrations by parts of the action, and letting the variations vanish at spatial infinity, we find that if the equations of motion are satisfied, then

$$\frac{\partial}{\partial t}\int d^3x\left(P\dot{\Phi}-p^a_{,a}\Phi\right)=-\int d^3x\partial_a j^a\Phi+\int d^3x j^0\dot{\Phi} \tag{10.36}$$

or

$$\int d^3x\left[P\ddot{\Phi}+\left(\dot{P}-p^a_{,a}-j^0\right)\dot{\Phi}-\left(\dot{p}^a_{,a}+j^a_{,a}\right)\Phi\right]=0$$
$$=:\int d^3x\left(g_2\ddot{\Phi}+g_1\dot{\Phi}+g_0\Phi\right). \tag{10.37}$$

Thus, the Gauss's law constraint $C := p^a_{,a} + j^0 = 0$ may be a consequence of the required vanishing of the time derivative of the primary constraint $P = 0$.

Chang showed generally that the consistency conditions (10.37) guarantee that if the equation of motion

$$\frac{\delta L}{\delta Q}=0 \tag{10.38}$$

is fulfilled at the initial time, then it is fulfilled for all time. For the electromagnetic example, this means that if

$$\frac{\delta L}{\delta Q}=\vec{\nabla}\cdot\dot{\vec{A}}-\nabla^2 Q+j^0=0 \tag{10.39}$$

is fulfilled at an initial time, then it will be fulfilled at all future times. Chang therefore proposes that the variables $Q := A_0$ and P can be eliminated entirely by setting $Q = 0$, and therefore setting as an initial condition

$$\vec{\nabla}\cdot\dot{\vec{A}}=-j^0. \tag{10.40}$$

Finally in passing to the quantum theory, he imposes this classical initial condition as a condition on the quantum state Ψ,

$$(\vec{\nabla}\cdot\vec{p}-j^0)\Psi=0. \tag{10.41}$$

So the ultimate practical outcome of this analysis for quantum electrodynamics is that Chang provided another, somewhat more general proof of the legitimacy of the second quantization procedure proposed by Heisenberg and Pauli (1930).

10.4 Conclusion

In summary, we have identified the following five original contributions of Chang to the quantization of constrained systems:

1. He was the first to recognize in print, and to offer a preliminary resolution of, a problem for quantization posed by the appearance of arbitrary space-time functions in classical gauge theories.
2. Chang proposed a procedure for imposing quantum constraints, using Lagrangian multipliers, for a limited class of non-singular classical theories.
3. He recognized that constrained systems could exist in which the constraints could be solved, thereby entirely eliminating some canonical degrees of freedom. Canonical Poisson bracket relations of the remaining phase space variables could then be replaced by canonical quantum commutation relations. Furthermore, he showed that in these models, the functions that multiply the constraints become functions of the dynamical variables, and that they therefore become non-trivial operators in quantum theory. Dirac later discovered that these models possessed the property that the Poisson brackets of the constraints among themselves did not vanish.
4. He discovered for a limited class of classical gauge theories that the preservation of primary constraints under time evolution leads to additional constraints. This discovery has until recently been attributed to Anderson and Bergmann in 1951, even though the general proof was already demonstrated by Rosenfeld in 1930.
5. Chang developed a technique for quantizing a class of gauge covariant models that could be viewed in the case of electromagnetism as a somewhat more general proof of the legitimacy of the quantization procedure proposed by Heisenberg and Pauli in 1930. This method formulates the gauge condition as a restriction on initial quantum states.

There is a clear link between Chang's activities during 1945–1947 and Dirac's subsequent publications beginning three years later on constrained Hamiltonian systems. In future work, we intend to further investigate the detailed manner in which they influenced each other.

Abbreviations and Archives

Niels Bohr Archive	Bohr Scientific Correspondence, Fowler to Bohr. Folder 19. No. 380608
Cambridge University Reporter	Reporter issues for the academic year 1936–1937, Vol. 67; 1937–1938, Vol. 68

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