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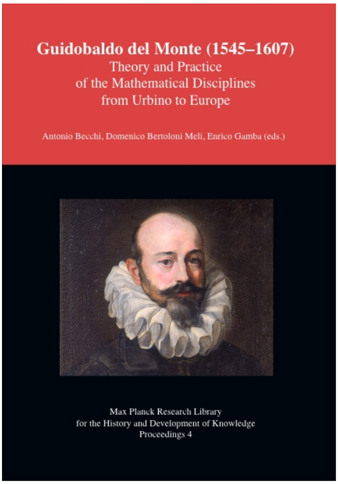

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Chapter 1
"Argumentandi modus huius scientiae maxime proprius." Guidobaldo's Mechanics and the Question of Mathematical Principles

Maarten Van Dyck

1.1 Introduction: Guidobaldo, Galileo, and the Subalternate Sciences

How should we place Guidobaldo del Monte in the changing landscape of late sixteenth, early seventeenth-century knowledge? At once a faithful Aristotelian and early patron of Galileo, Guidobaldo seems to defy some naïve conceptions about the nature of the so-called scientific revolution.[1] As is well known, one of the ways in which the mathematician Guidobaldo can be considered to have been a faithful Aristotelian is exactly that he is almost completely silent on philosophical issues, thus respecting the disciplinary boundaries that had become deeply engrained in the fields of knowledge. But this leaves us with little to go on if we want to ascertain how he would have understood his own endeavors, and possibly what connected them to or separated them from those of his younger friend, Galileo.

In the present paper I will try to argue for what I take to be a crucial connection between Guidobaldo's work on mechanics and Galileo's aspirations in constructing a new, mathematical science of nature. In particular, I will analyze how Guidobaldo laid bare the conditions under which Archimedes's *mathematical* science of mechanics could be established, and argue that this kind of focus on the conditions that allow for mathematization implicitly prepared the way for Galileo's new philosophy of nature. It is thus a deliberate engagement with problems posed by the mathematization of phenomena that really allows us to see

[1] Bertoloni Meli (1992) gathers sufficient evidence to believe that Guidobaldo would have seen himself as a faithful Aristotelian. In what follows I will add some further evidence, when considering Guidobaldo's views on mathematical abstraction as applied to mechanical concepts. In the conclusion, however, I will offer some reflections that could make us somewhat more careful in our use of such a denomination—as opposed to the question of how Guidobaldo would have seen himself—by pointing in which way he ignored important Aristotelian epistemological constraints, and in doing so helped prepare the way for a thoroughly anti-Aristotelian philosophy of nature.

Galileo following in the footsteps of Guidobaldo, notwithstanding their differences of opinion on where this path would lead.[2]

Seeing the connection between Galileo and Guidobaldo in these terms is not unconnected with the claims that have been made for the importance of the category of the subalternate (or middle or mixed) sciences.[3] Since the 1970s, a number of historians of science have pointed out that Galileo's "new" sciences had many of the characteristics of these sciences (which apply mathematical demonstrations to physical subject matter) as they were discussed within Aristotelian philosophy.[4] As a result, we have at least been made aware of the fact that it was possible to discuss and integrate some aspects of the applied mathematical sciences within an Aristotelian framework. It is also undeniable that these debates are extremely relevant if we want to understand the predicament of for example the Jesuit mathematicians throughout the first half of the seventeenth century as they tried to enter into critical discussion with (and possibly contribute to) the recent developments in the mathematical sciences without leaving what could be considered to be a general Aristotelian framework (Dear 1995; Feldhay 1999). But there has also been some apparently well-founded scepticism which centers on whether these same debates really can teach us something important about Galileo's own position. One of the most outspoken criticisms came from Laird, who was actually one of the first to stress the potential importance of the category of the subalternate sciences in the early 1980s. In a paper from 1997, however, he attacked the importance of the category for understanding Galileo by pointing out that Galileo does not even mention the existence of the category in places where we would have him expected to do so. Moreover, and more importantly, Laird also notices that the philosophical discussions surrounding this category could not have appealed to Galileo at all, since the Jesuit teachings with which he was definitely familiar were, to say the least, rather sceptical about the proper scientific status of the subalternate sciences. Finally, and this is probably the most impor-

[2] I will not comment on the significant difference of opinion between Guidobaldo and Galileo on the conditions under which the phenomenon of *motion* could be legitimately mathematized. This difference has already been often commented upon, but in my opinion many of these comments suffer from an insufficient understanding of Guidobaldo's quite sophisticated understanding of the problems involved. This paper can thus be seen as laying part of the groundwork that, by stressing the important convergence of thought on a number of issues, should make possible a much more nuanced understanding of what in the end separated Guidobaldo and Galileo.

[3] In this paper, I will use "subalternate" consistently. Although usage among the Aristotelian commentators is not fixed, it is important to notice that "mixed" (or its cognates) only seems to have come into use in the seventeenth century (none of the sixteenth-century mathematical or philosophical authors that I am aware of use it), and as this might reflect some important changes in the understanding of the most fundamental characteristics of these sciences—as I will indicate in the concluding section of this paper—I believe it should be avoided when discussing the earlier incarnations of the notion.

[4] See (Machamer 1978; Wallace 1984; Lennox 1986; Biener 2004).

tant argument, he shows that these philosophical discussions would not even have answered any questions that Galileo found pressing[5] (we will come back to the reason why in Section 1.2).

But if we grant this conclusion with respect to Galileo, what does this tell us about his connection to the enterprise of people like Guidobaldo? One of the major motivations for linking Galileo to the subalternate sciences has always been the idea that he stood in a well-established sixteenth-century tradition, and that his new sciences thus did not arise miraculously. This idea also lies behind a very thoughtful recent reconsideration of Galileo's relation to the subalternate sciences by Biener (2004), who claims that Laird's argument shows we should distinguish between the subalternate sciences as *a philosophical tradition* and as *a practical one*. And whereas it might well be that the philosophical tradition could not have appealed at all to Galileo, the practical tradition nevertheless appears to have provided the model according to which Galileo did structure his own new (mathematical) sciences—as Biener's subtle analysis of the first two days of the Discorsi shows. The present paper can be seen as a further elaboration of this perspective by investigating the approach of the mathematical practitioner with whom Galileo might most reasonably have been expected to share a tradition: Guidobaldo del Monte. This will also allow us to sharpen the distinction made by Biener by providing an understanding of how exactly the practical tradition provided some of the methodological insights that someone like Galileo was looking for, and which the philosophical tradition conspicuously missed.

Part of my conclusion will indeed be that Laird's analysis regarding Galileo's indifference for the philosophical discussions about the subalternate sciences can be carried over almost completely to the case of Guidobaldo.[6] At first sight, this might be surprising since, as already mentioned, Guidobaldo is in many respects rightfully portrayed as a faithful Aristotelian. Nevertheless, the main reason why he did not engage directly with the philosophical discourse surrounding the category is that it would not have been helpful in answering the questions regarding mathematization which Guidobaldo *as a practitioner* of the mathematical sciences found pressing—rather that it would even have confused issues in an important way! I hope to show that the way in which he *did* answer these questions, moreover, *does* show us an important and very interesting link between Guidobaldo's and Galileo's work.

The structure of my paper is as follows. I will first give a quick overview of the concept of the subalternate sciences as it was being discussed by philoso-

[5] See (Laird 1997).

[6] In an earlier paper (Van Dyck 2006), I include Guidobaldo's mechanics within the category of mixed sciences, without much ado. I do believe that most claims in that paper still stand, but that there are good reasons to be more careful with the use of the category.

phers from Grosseteste onwards. I will then discuss how this concept was being applied to the sixteenth-century science of mechanics, also by the practitioners themselves. Given what I have already said about the conclusion of my paper, this might seem to introduce an uncomfortable tension in my argument, but it will turn out that the tension actually lies in the application of the category itself. This will become more apparent if we next move to Guidobaldo, and especially to his commentary on the Archimedean treatise on the equilibrium of planes. From there on, I will turn to a more detailed analysis of Guidobaldo's commentary of Archimedes's proof of the law of the lever. It will turn out that it is here that we can find the elements of a tradition in which we can place Galileo in what I take to be a really *illuminating* way. In elaborating this point, I will then come back to the distinction between a practical and a philosophical tradition of subalternate sciences.

1.2 A Quick Tour of the Subalternate Sciences

The present section will be kept to an absolute minimum as good discussions already exist. I base my summary mainly on (Laird 1983), to which I refer for all detailed references (Laird 1997 also contains a neat summary).

Puzzled by some of Aristotle's remarks in the *Posterior Analytics* about some sciences "being under" other sciences, commentators on the Stagirite's work elaborated and analyzed the category of the "subalternate" sciences. The context in which Aristotle had introduced the germs of this idea was in discussing how some sciences could use mathematical demonstrations to arrive at conclusions about physical things, apparently violating the essential Aristotelian requirement of homogeneity which states that all terms in valid scientific demonstrations must belong to the same genus. Nevertheless, sciences like astronomy or optics can use mathematical principles, he claimed, because they are related to mathematics as "one under the other." He moreover added that the lower science (e.g., optics) knows the fact (*oti, quia*), while the reasoned fact (*dioti, propter quid*) belongs to the higher science (e.g., geometry). In *Physics* Aristotle also called astronomy, optics and harmonics the "more physical of the mathematical sciences" (to add to the confusion, in the oft-used translation of the Middle Ages by James of Venice, this is translated as "more physical than mathematical"). While in geometry one treats physical lines as mathematical rather than physical, in optics one treats mathematical lines as physical rather than mathematical. In *Metaphysics*, finally, there is a passage where Aristotle makes the seemingly contrary claim that optics treats visual rays (i.e., physical lines), but only as mathematical lines.

It is clear that this is a rather scant basis on which to develop a full-fledged theory, moreover leaving ample room for disagreement, not least due to the apparent incoherence in Aristotle's own statements. But the demands on such a theory were more or less clear: it should explain *what* the objects of the subalternate sciences are, *how* these objects are considered, and how this enables us to understand what is *quia*, and what is *propter quid* about the different demonstrations in which these objects figure. On the first question, there seems to have been general agreement that the subalternate sciences deal with some kind of composite subject which is basically a mathematical object to which an extra sensible condition is added (such as "visual" to line). The second and third questions received very different answers. It was both thought that the subalternate sciences basically consider these objects as somehow physical (e.g., by Grosseteste), and that they consider them purely as mathematical (e.g., by Zabarella). Under the influence of Averroes, later commentators also phrased this as the relation between the *resconsiderandi* (which according to Zabarella, for example, is the added *sensible* condition) and the *modus considerandi* (which again according to Zabarella is *mathematical*). The answer to this second question obviously influenced the widely differing interpretations of what exactly was demonstrated *quia*, and what was known *propter quid*.

I do not think it is necessary to enter here into any of the further details (although these are important in that they remind us that there was not just one Aristotelian position on these issues). For now, I only want to draw attention to a fact that is already forcefully stressed by Laird in his paper attacking the relevance of the category for understanding Galileo. We should be very clear about the rather limited nature of the basic issue that the concept was thought to address. In Laird's words, the question the commentators tried to answer was what "demonstrative force a purely mathematical demonstration retained when […] applied to a physical subject" (Laird 1997, 259). Translated in the traditional Aristotelian syllogistic framework, the issue was the following: given that a minor premise attributes a mathematical property to a physical object (e.g., visual rays having a certain angle), the question was whether the major premise showing further mathematical properties to follow from this basic property could retain its full demonstrative force, that is, whether the middle term linking minor and major could indeed function as a middle (whether one could interpret the terms in the minor in their full mathematical meaning). What was never in question in these discussions was whether and how it is possible to establish the minor premise. That is, it is simply assumed that natural bodies have certain quantitative aspects.[7] When discussing, for example, the proof of the law of reflection in optics, the (empirical) principle stating that certain properties hold between the

[7] This is the third reason adduced by Laird (1997) for doubting the usefulness of the category for Galileo's new sciences (cf. Section 1.1 of the present paper). The most pressing problem that confronted Galileo was exactly the establishment of such "mixed" premises.

angles of incoming and reflected rays is simply assumed without further comment—the question is whether one can use further geometrical principles about triangles to analyze some of the consequences following from this principle.[8]

1.3 The Scope of Mechanics in the Sixteenth Century

Again, I can be rather brief because the broad lines of the story are already rather well known. I will be mainly interested in stressing two considerations that are not always appreciated as clearly as they should be, but that are relevant for understanding the prospects of mechanics as a subalternate science. They should help us to better appreciate Guidobaldo's complete silence on the issue of subalternation.

The title of the present section obviously refers to the influential paper by Laird , which helped to sketch the contours of the landscape of sixteenth-century views on the nature of mechanics as a legitimate scientific discipline (Laird 1986). The major event in this process of legitimization, as retold by Laird, was the rediscovery of the (pseudo-)Aristotelian *Mechanical Problems*. The mere existence of a treatise on mechanics thought to be written by Aristotle was in itself already a major factor in this legitimization, but the preface to that treatise also contained some *topoi* that could be fruitfully exploited to strengthen this legitimacy. Laird summarizes these as follows:

> first, it [mechanics] was a theoretical science rather than a manual art; second, it was mathematical, although its subject was natural; third, it concerned motions and effects outside of or even against nature; and fourth, it produced them for human ends. (Laird 1986, 45–46)

Let us try to focus a little more on the second and third aspects. The second aspect (a mathematical science of a natural subject) of course immediately brings the subalternate sciences to mind, especially as the Aristotelian preface also claims that "the how" of mechanical problems is known through mathematics, and "the about what" through physics. Thus, it is not surprising that most sixteenth-century commentators on the *Mechanical Problems* explicitly linked this to the philosophical discussions on the subalternate sciences referred to in the previous section. They also agreed that mechanics considered its subject matter through mathematical considerations. Tartaglia and Baldi spoke about mechanics as a *scientia subalternata*; Maurolico noticed that mechanics was a *scientia me-*

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[8] One can see this, for example, in Grosseteste's discussion of this proof, analyzed in (Laird 1983, 37).

dia between the mathematical and the natural (Tartaglia 1546, 82v; Baldi 1621, 4; Maurolico 1613, 7–8).[9]

So far, this is the familiar story that clearly links important practitioners, such as Tartaglia, Baldi and Maurolico, to the tradition of subalternate sciences. However, a potential complication arises when we try to see how this can be squared with the third aspect singled out by Laird: that mechanics treats effects *praeter naturam*.[10] As with the second aspect, this could also be fruitfully linked to other places in the Aristotelian corpus. After all, the basic opposition between what is according to nature and what goes counter to it is one of the true cornerstones of Aristotle's philosophy. Moreover, the discussion in *Physics II* concerning the distinction between the natural and the artificial is obviously relevant to the case of mechanics and was accordingly often referred to. In the passages discussing this distinction, Aristotle famously claimed that art imitates nature, which was a statement that was frequently taken up by the commentators on the *Mechanical Problems*. The basic Aristotelian idea is that art does not simply overrule nature, but that it profits from the natural constitution of things to bring about useful effects that would not arise naturally. Moreover, it brings about these effects by imitating nature. As Aristotle claims: "if a house, e.g., had been a thing made by nature, it would have been made in the same way as it is now by art" (the underlying idea apparently being that art takes all its clues from nature's characteristic ways of organizing matter in goal-oriented ways) (Aristotle 1930, 199[a]). Guidobaldo was especially explicit when he stated that the arts are able to bring about effects that are *praeter naturam* exactly *because* they imitate nature, but this idea seems to have been generally shared by most commentators.[11]

But this could be thought to leave us in a quandary. If it is true that art imitates nature, how could it be that mechanical demonstrations (supposedly explaining something about how art can achieve its goals) are wholly mathematical: nature certainly does not operate according to mathematical principles on an Aristotelian view. I do not want to overstate this point, or to make too much out of it, but it does seem that the second and third aspect identified by Laird do not sit very easily together. And I do believe that this helps explain why Guidobaldo, who opens his 1588 *Paraphrasis* with an extended discussion of art's imitation of nature, does

[9]Both the works of Baldi and Maurolico were written in the sixteenth century, but only published posthumously.

[10]There were many ways in which this opposition was phrased, but the expression *praeter naturam* seems to have been most used, and the one to express the generally accepted line of thought on the issue most accurately. See (Festa and Roux 2001) for further references and for some of the issues surrounding this crucial aspect of the mechanical sciences. Popplow (1998, 154–168) also offers some further discussions on the theme.

[11]For more detailed discussions, see (Monte 1588, 2; Piccolomini 1565, 7v; Maurolico 1613, 29; Monantheuil 1599, 8). The next section will analyze Guidobaldo's particular interpretation.

not refer to the idea of subalternate sciences. I hope to make this claim more plausible in the next section, but before doing so, I would like to add a further consideration that is relevant to this issue.

The story about the legitimate scope of the mechanical sciences in the sixteenth century has often concentrated on the reception of the *Mechanical Problems*, of course coupled with the publication of the Archimedean treatises. Because of their highly abstract mathematical character, however, the latter do not seem to contain much material that is directly relevant to defining this scope. The goal of the next two sections will be to counter exactly this impression, but it is equally important to stress that writers on mechanics had more sources at their disposal than just these two in crafting an interesting image for their science—sources that often offered a much more encompassing vision of the scope of mechanics. Vitrivius work was of course directly relevant, and by the second half of the century the eighth book of Pappus's *Mathematical Collections* should also be added to the available classical background. The work was only published in 1588, but Guidobaldo knew its contents much earlier through his association with Commandino, who was responsible for the translation. It is thus not accidental that the introduction to his *Mechanicorum liber*, and especially the dedicatory letter by Filippo Pigafetta to its Italian translation of 1581, contain much more substantial allusions to Pappus's introduction of his eighth book than to the preface to the *Mechanical Problems* (Monte 1577; 1581).[12] This is not only relevant because Pappus introduced the idea of systematizing a science explicitly devoted to the five simple machines, which could all be explained from a common principle (an idea for which he refers to Heron and which would be taken up by Guidobaldo in his *Mechanicorum liber*), but also, and more importantly for our topic, because he explicitly stated that the theoretical part of mechanics makes use of "geometry, & arithmetic, & astronomy, & physics!"[13]

Not only was there a potential tension between seeing art as imitating nature and considering mechanical demonstrations to be purely mathematical, there was also an alternative authoritative view that included physical considerations within the theoretical part of mechanics! We can thus have Baldi stating on one page that mechanics as a subalternate science treats physical subjects with geometrical demonstrations, and then have him switch, apparently unproblematically, to the statement on the next page that its demonstrations use geometrical, arithmetical, and physical considerations (Baldi 1621, page 4–5 of the unnumbered preface). The least we can say is that Baldi does not seem to be concerned about the philo-

[12] Pigafetta wrote his translation in close association with Guidobaldo, so we can safely assume the almost literal extracts from Pappus to have been provided by Guidobaldo.

[13] In the translation by Commandino: "rationalem quidem partem ex geometria, & arithemetica, & astronomia, & ex physicis rationibus constare" (Pappus 1660 [1588], 447).

sophical subtleties involving the subalternate sciences. We will see in the next two sections that Guidobaldo, who paid painstaking attention to the structure of the Archimedean demonstrations, had positive reasons to neglect these subtleties and to favor instead the characterization taken over from Pappus.

1.4 The Mathematical and the Natural in Guidobaldo's Paraphrasis

After having published his influential *Mechanicorum liber* in 1577, which as already mentioned saw its Italian translation in 1581, Guidobaldo in 1588 also published a detailed commentary on Archimedes's *Equilibrium of plane figures* (Monte 1577; 1588). As Guidobaldo explains in his dedicatory letter, *Paraphrasis* is meant to answer criticisms that were made of his earlier *Mechanicorum liber* by people who were maybe not so adept in "the mechanical way of investigating the causes of things" (Monte 1588, page 1 of the unnumbered dedicatory letter). In this way, he immediately introduces one of the running themes of his commentary—if not the most important message of the whole book—that the mechanical science has a special way of demonstrating its propositions, which must be grasped before one can truly understand any of its claims. In *Mechanicorum liber* Guidobaldo had moreover *assumed* the validity of the law of the lever, whereas this law actually contains the true foundation of the science of mechanics (as he never tires of stressing in this work devoted to the demonstration of its validity).[14] In *Paraphrasis*, Guidobaldo accordingly shows that the validity of the law of the lever is indeed grounded in a special method of demonstration—in the "argumentandi modus huius scientiae maximè proprius [...]" (Monte 1588, 44)—implying that whoever does not grasp this method of demonstration actually ignores the proper foundations for the mechanical science.

The opening pages of the preface to *Paraphrasis* are much more explicitly tied to the preface to the Aristotelian *Mechanical Problems* than was the preface to *Mechanicorum liber*. This means that Guidobaldo in turn touches on the admiration accorded to mechanical effects, and especially the link of this admiration with their *praeter naturam* character, and on the mechanical science having both mathematical and physical characteristics. His discussion of the *praeter naturam* effects is concise and very elegant. He begins by recalling that Aristotle in *Physics II* and the *Mechanical Problems* had considered three ways in which art can operate: by imitating nature (such as in sculpture), by finishing what nature could not achieve (such as in medicine), and by operating *praeter naturam* (such as in mechanics). But, he adds, on closer consideration it turns out that the latter

[14] In proposition V of *On the balance*, the first book of *Mechanicorum liber*, Guidobaldo at first sight does give a proof of the law of the lever, but he actually assumes its validity also in the proof of that proposition.

also happens through the imitation of nature, since "it is through nature itself that nature is overcome." This can be made clear as follows, he explains. Suppose we have two bodies, A and B, of which A is heavier than B. It would be in the nature of things that A would be able to raise B but not the other way around. Consider however what happens if we put them on some lever in such a way that their common center of gravity C lies in-between B and the fulcrum D which by its nature can not move: the center of gravity will by its own nature move down, which will have the effect that (because of the presence of the fixed point D) A will be raised and B will be lowered. So what is it that art brings about? Nothing other than that it places things with respect to each other in such a way that thereupon the desired effect follows by just letting nature follow its course (Monte 1588, 2–3).

The basic explanatory scheme behind this discussion is of course the one expounded in great detail in Guidobaldo's *Mechanicorum liber*, which explains the operation of all simple machines through the relative positions of a compound system's center of gravity and a fulcrum.[15] Let us just note here how this allows Guidobaldo to make much better sense of the way in which *overcoming* and *imitating* nature are inextricably interwoven. This becomes especially clear if we contrast it with the difficult-to-grasp Aristotelian identification of the *praeter naturam* aspect of mechanical phenomena with the part of circular motion that is supposedly *praeter naturam*.[16] But this also means that his way of dealing with this classical topos has only aggravated the tension with the second aspect that was identified by Laird in the Aristotelian preface: the essentially mathematical character of mechanical explanations. And this was of course a problem that did not arise in the same way for the vague Aristotelian explanation of the *praeter naturam* effect, since this was actually grounded in the *mathematical* nature of the circle!

In a passage immediately following the explanation of the *praeter naturam* nature of mechanical phenomena, Guidobaldo enters into the physical/mathematical issue, thus following the broad lines of the structure of the Aristotelian preface, but he does so in a rather unexpected way which sets the stage for the rest of his commentary. He reports that Aristotle in his preface considered mechanics to partake both of the mathematical and the natural and that Archimedes was obviously familiar with these views, because he considers mathematical things, such as distances and proportions, through geometrical demonstrations; and because he considers natural things through natural considerations, such as those relating to the nature of the center of gravity and motion up and down (Monte 1588, 4–5).

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[15] For more details on this scheme, see (Van Dyck 2006).

[16] Aristotle had claimed that all circular motion is composed of a natural and a praeternatural component. For a detailed analysis, see (Micheli 1995, 41–86).

In itself this is not a very enlightening statement, but we will see in the next section that it actually refers to a subtle understanding of the structure of Archimedes's treatise and its proof procedures. For now, let me just note how even this seemingly casual statement makes it rather hard to see how this could be squared with the category of subalternate sciences, which demands that the subject matter is treated either mathematically or physically (this condition is the only way to guarantee that the requirement of homogeneity is not violated); whereas it does explain why Guidobaldo would have been naturally attracted toward Pappus's more liberal characterisation of the nature of mechanical demonstrations.

It must be noted that Guidobaldo had also opened his *Mechanicorum liber* with a discussion of mechanics operating against nature, in which he claimed that mechanics comes from the union of geometry and physics. But at this point he did seem to understand this union more or less along the lines of the philosophical discussions on the subalternate sciences, as is testified by the fact that he goes on to identify (implicitly) the physical part with the material substrate and the geometrical part with necessary demonstrations. He then further claimed that mechanics exerts control over physical things (presumably because it is understood to apply necessary demonstrations to physical subject matter), by operating against the laws of nature. His short discussion is ended by claiming that mechanics "adversus naturam vel eiusdem emulata leges excercet" (Monte 1577, page 2 of unnumbered preface). This last characterization was translated by Stillman Drake as "[it] operates against nature or rather in rivalry with the laws of nature" (Drake and Drabkin 1969, 241), which is certainly a possible translation. What is obscured in rendering it thus, however, is that the semantic field of "aemulatio" also includes clear notions of imitation besides rivalry (Glare 1996);[17] but it must be admitted that this link is not yet brought out explicitly by Guidobaldo in this discussion. It thus seems that when writing the preface to his *Mechanicorum liber*, Guidobaldo had not yet really thought through how to understand this operation against, or possibly in imitation of, nature. This silence allowed him to introduce a clear echo of the characterization of subalternate sciences in discussing the *praeter naturam* nature of mechanics. It is the conceptualization of mechanical phenomena as introduced in *Mechanicorum liber* itself, however, that provides the clue to understanding this operation in the passage from *Paraphrasis* discussed above. And at this point, "holding control over physical things" no longer comes about by the simple application of geometrical arguments to physical subject matter, but by exploiting some of the physical properties of this

[17] In a famous definition, Cicero clearly links "aemulatio" with "imitation:" "imitatio virtutis aemulatio dicitur" (Cicero 1971). A further interesting occurrence carrying a clear link to imitation is in Claudianus's poem on Archimedes's planetarium, which is cited by Henri Monantheuil in his commentary on the *Mechanical Problems* (Monantheuil 1599, 3–4 of unnumbered preface).

subject matter in a cunning way—by also approaching "natural things through natural considerations, such as those relating to the nature of the center of gravity and motion up and down."

Let us now return to Guidobaldo's exposition of the Archimedean treatise. Before entering into the propositions and their proofs, Guidobaldo deems it necessary to explain two further things in his preface. First it needs to be understood what the proper definition of center of gravity is, and secondly it needs to be explained how Archimedes can treat the center of gravity *of plane figures*.

The absence of a definition of center of gravity in Archimedes's treatise is solved in exactly the same way as in the earlier *Mechanicorum liber* by introducing the definition that Pappus had given to the notion (which is obviously a further token of the great importance of Pappus's treatise in shaping late sixteenth-century views on the scope of mechanics). This definition reads:

> The centre of gravity of any body is a certain point within it, from which, if it is imagined to be suspended and carried, it remains stable and maintains the position which it had at the beginning, and is not set to rotating around that point. (Monte 1588, 9)[18]

Following the example of Pappus himself, Guidobaldo immediately links the existence of such a point within any body with the natural propensity that all bodies have to go to the center of the world. This essential connection is understood if we consider what happens with a body that is hypothetically placed at the center of the world: since it will be absolutely at rest, there must be a point in the body around which all parts of the body have "equal moment" (in phrasing the condition in this terminology, Guidobaldo refers to the alternative definition as given by Commandino, the first part of which states that "the centre of gravity of any solid shape is that point within it around which are disposed on all sides parts of equal moments") (Monte 1588, 9).[19] This is actually the only place where Guidobaldo actually uses the notion of "moment." This situation indeed presents a peculiar configuration (which forced Guidobaldo to switch to a more abstract way of characterizing the situation): the weights are actually weighing along the arms of the balance. This is a kind of cosmological singularity which will later also plague the geostatic debate in the 1630s in France. It brings to mind what Dijksterhuis says in the Archimedean hydrostatical proofs about the peculiar role played by the center of the world (Dijksterhuis 1956, 377–379). The center of the cosmos in both contexts plays the role of absolutely resisting all forces without in itself being a physical point attached to something (if Mersenne had thought this

[18]Translation from (Drake and Drabkin 1969, 259).
[19]Translation from (Drake and Drabkin 1969, 259).

through he undoubtedly could have constructed a beautiful proof of the existence and nature of God). In the end, it points to the difficulty of really building a satisfactory abstract theory of weight *within* a cosmological frame. As long as weight is something internal to bodies, these kinds of problems are bound to crop up, and it is thus this point in which we must think the natural propensity (which gives rise to a body's weight) to be concentrated—because it is actually this point which truly wants to unite itself with the center of the world.

Guidobaldo next introduces a further consideration which is relevant to the relation between mathematical and physical notions (Monte 1588, 11). He states that besides the center of gravity, we can introduce three further centers in our considerations: the center of a figure, the center of a magnitude, and the center of the world. The distinction between the first two is not immediately relevant to our purposes, for which it is only important to notice that they are both mathematical notions. Guidobaldo then considers different scenarios in which all four centers either coincide or differ in different combinations. In the first scenario all centers coincide, which is the case if we consider the earth. This is of course a significant example. The fact that the earth is supposed to have a mathematically determinate spherical form was one of the staple examples in debating the nature of mathematical abstraction in the Aristotelian tradition, and Guidobaldo introduces it by stating that it is acknowledged by everybody. He further refers to Aristotle's *De caelo* and Archimedes *On Floating Bodies* for the fact that the center of the earth's circular form coincides with the center of the world; to which he further adds that the stability of the earth in this position implies that the earth's center of gravity also coincides with these three centers. The discussion of this kind of exemplary scenario brings out the essential dual nature of a body's center of gravity. It is a notion which can be ascribed to every physical body having a natural tendency for motion, but which at the same time is to be connected with some of the mathematical accidents of this body, such as its geometrical form and position. It is this double aspect that lies behind Guidobaldo's earlier quoted assessment that Archimedes's considered mathematical things, such as distances and proportions, through geometrical demonstrations; and that he considered natural things through natural considerations, such as those relating to the nature of the center of gravity and motion up and down. What is clarified through this further discussion is that the notion of center of gravity essentially binds together both kinds of considerations. It is connected with physical properties, such as the equilibrium effects of weight, but at the same time it is to be considered as a mathematical point which can thus be introduced in geometrical demonstrations: just as we can abstract the geometrical spherical form of the earth from its physical makeup, so we can also abstract the geometrical point in which its physical center of gravity is situated. In one of the crucial scholia in which Guidobaldo discusses the Archimedean proof method he expands a bit further on this double nature (Monte 1588, 48–49). He stresses that insofar as the notion is introduced in geometrical proofs, it is to be ascribed to bodies considered as geometrical magnitudes, but that insofar as it is linked directly with effects of equilibrium, it is to be ascribed to these same bodies as heavy. But, he hastens to add, also when we consider bodies as magnitudes, we have to understand that we are dealing with magnitudes *to which heaviness is predicated*—as otherwise the notion of center of gravity would lose all meaning.

In passages like these, Guidobaldo pays considerable attention to the possibility of inscribing mechanics in an Aristotelian framework, with its sophisticated understanding of abstraction as a mental operation, and in doing so, comes close to certain positions defended in the debates on the status of the subalternate sciences. Such an understanding of mathematical abstraction was indeed one of the main factors that lay behind the different views on the relation between subalternating and subalternated sciences (Laird 1983). The main idea was that pure mathematics arose on the basis of the abstraction of quantitative properties from physical bodies; an abstraction that consisted in considering these properties as if they were not in sensible matter (which in reality they are). A science subalternated to mathematics then applies these abstracted properties back to natural things; an application which was understood as the addition (or predication) of a sensible condition, such as weight, to the mathematical magnitudes.

It is thus to the extent that Guidobaldo wants to do justice to the Aristotelian theory of abstraction that his pronouncements fit very nicely with the philosophical debates on the status of subalternate sciences. But it must be stressed that nowhere does he connect this with the further issue of the status of the resulting demonstrations, nor does he claim these demonstrations to be essentially mathematical or physical—a point to be expanded upon in the next section. That is, the ontological status of mathematical entities is a possible point of concern in his mind, but the epistemological requirement of homogeneity (which, one will recall, was the main impetus behind the debate on the subalternate sciences) does not seem to attract his attention.

The fact that we must always consider geometrical magnitudes to which the property of weight is predicated raises a further problem on which Guidobaldo comments in his preface (Monte 1588, 15–18). Indeed, if this is the case, it becomes hard to understand how Archimedes's claim to offer a treatment of the equilibrium of *plane figures* could be well founded, since, as Guidobaldo puts it, such a predicate is completely alien to the nature of plane figures. He tries to dismantle this objection by admitting that insofar as they are plane figures, they indeed have no weight at all, but by stressing that we can still "mentally conceive" plane figures to be equilibrating and thus showing the effects of gravity. He offers three reasons for this view. Firstly, he explains that we can consider any plane figure to be the upper surface of a solid body that is suspended in equilibrium, upon which we can conclude that this plane figure is also in equilibrium. We can moreover designate exactly one point in that surface as the center of gravity of that plane figure, as there will only be one such point from which when suspended the solid will remain at rest. There is thus nothing contradictory in also ascribing a center of gravity to plane figures.[20] Secondly, he asks why it would be legitimate for a mathematician to consider heavy bodies as if they had no weight (which is a point generally acknowledged by Aristotelian philosophers), whereas it would not be legitimate to consider things that have no weight as if they did have it. And thirdly, he remarks that we can easily imagine that a greater plane figure represents a greater weight than a smaller figure, and that we thus can also imagine these plane figures to be in both equilibrium and disequilibrium.

Maybe the most important of Guidobaldo's considerations, however, is that the first eight propositions, which contain the true foundation of the science of mechanics, need not be limited to plane figures (Monte 1588, 19). Guidobaldo thus stresses that Archimedes in these propositions refers to "magnitudes" in general, as this name is common both to solid and plane figures (Guidobaldo even goes as far as having his figures accompanying these propositions alternatively represent solid and plane figures). The restriction to plane figures is thus actually only relevant to the further propositions, introduced to square the parabola, and need not concern anyone who is primarily interested in understanding the foundations of the science of mechanics.

1.5 Demonstrating the Law of the Lever

It was already mentioned how Guidobaldo throughout his preface and commentary stresses the fact that Archimedes's first eight propositions (i.e., up until the law of the lever) provide the true fundaments, or the "elements," of the science of mechanics. It is of course not hard to see why: it is only this law that offers a precise mathematical determination of the conditions for the equilibrium of any balance, and by extension (as demonstrated earlier in *Mechanicorum liber*) the conditions of equilibrium for any simple machine (and thus the ensuing multiplication of force). Guidobaldo's commentaries on the proof of this law accordingly focus on the question of how it is possible to give such a *mathematical* deter-

[20] Luca Valerio would also pay much attention to this problem, more or less taking over Guidobaldo's solution (Napolitani 1982). Baldi also mentions the problem, and again introduces the idea that plane figures are the surfaces of solids. This line of thought implies that the center of gravity of solid figures is prior to that of plane figures, which thus added extra importance to the enterprise of Maurolico, Commandino, Galileo and Valerio to study this topic on which no extant Archimedean writings exist.

mination, and he painstakingly lays out what he sees as the crucial factor in this respect: the peculiar nature of a body's center of gravity as defined by Pappus. It is important to point out immediately that this definition is of a *purely physical* nature, and as such leaves its mathematical determination completely open. Giving such a precise determination is thus exactly the task of the first eight propositions of Archimedes's treatise. Guidobaldo's commentaries accordingly focus on precisely this problem: how does this purely physical characterization allow for a precise mathematical determination? In the preceding section we already saw part of the answer: as a point that is situated in a body, it is linked with some of the body's physical properties (tendency toward motion and equilibrium) but it can also be treated *as* a mathematical point. The important question now becomes: how is it possible to determine further mathematical properties of this point on the basis of nothing more than its physical nature?

At first sight, this might be thought to resemble the crucial question addressed by the philosophers discussing the status of the subalternate sciences—how it is possible to apply mathematical demonstrations to physical things characterized mathematically, so as to demonstrate further mathematical properties of these things? Such a similarity would only hold, however, if the determination of the further mathematical properties (the law of the lever) were based only on the center of gravity being a mathematical point (the only mathematical characterization given at this point), which decidedly can do no justice to Archimedes's demonstration. Whereas the philosophers wonder about the validity of mathematically demonstrating further properties of a thing to which is ascribed any mathematically property, Guidobaldo wants to show (with Archimedes) how it is possible to ascribe a *sufficiently rich* mathematical structure, which can then really ground a fruitful science. And as will becomes clear, this involves both physical argumentation and mathematical demonstration in a two-way interaction which is much too subtle to be grasped by a crude opposition between considering the subject matter *either* as physical or as mathematical.

A more interesting perspective on Guidobaldo's exposition of Archimedes's proof procedure comes to light if we connect it with Mach's well-known criticism of the same proof (Mach 1960, 13–32). Mach famously questioned how Archimedes could possibly determine the conditions for equilibrium of unequal bodies assuming as only input that equal bodies at equal distances are in equilibrium. According to him, the only way in which the inverse proportionally encoded in the law of the lever could be derived from the symmetrical situation was by actually presupposing its validity in the proof itself. The crucial point that he singled out for his criticism was the following:

> Archimedes makes the action of two equal weights to be the same under all circumstances as that of the combined weights acting at

> the middle point of their line of junction. But, seeing that he both knows and assumes that distance from the fulcrum is determinative, this procedure is by the premises unpermissible, if the two weights are situated at unequal distances from the fulcrum. (Mach 1960, 20)

Moreover, Mach adds that assuming it is permissible comes down to implicitly stating the law of the lever. In 1903, G. Vailati had already pointed out that Mach's criticism rested on the failure to see that the "unpermissible" assumption was in all probability grounded in the properties of a body's center of gravity (a notion that was not defined in Archimedes's treatise, but in all probability he had treated in other writings), and that this thus in no way comes down to presupposing the validity of the precise mathematical form of the law of the lever (Vailati 1987).

It is exactly this insight that Guidobaldo had already expounded at great length four hundred years earlier when he tried to explain how the physical properties of a body's center of gravity allow us to give it a precise mathematical determination. Moreover, Guidobaldo clearly recognized that the assumption singled out by Mach is indeed crucial in this respect. The fourth proposition of the Archimedean treatise occupies a central place in Guidobaldo's comments. This proposition states that

> if two equal magnitudes do not have the same center of gravity, then the magnitude that is composed of both magnitudes has its center of gravity in the middle of the line that connects the centers of gravity of the magnitudes. (Monte 1588, 42)

One of the comments Guidobaldo adduces to this proposition is that Archimedes in the earlier propositions (including the postulates) speaks about "gravia" whereas in this and the next propositions he speaks about "magnitudines" (Monte 1588, 48). This is of course intimately related to the double nature of the concept of center of gravity, already commented upon in the previous section. It is both linked with the heaviness of bodies, and the effects following from this property, and a mathematical point located in these bodies. The fact that it is in this proposition that Archimedes switches to mathematical terminology to denote the bodies thus signals to us that we will now enter into the mathematical determination of that point. As Guidobaldo notes, the proof of the fourth proposition itself depends on the second postulate (which states that if equal bodies are suspended at unequal distances, the one that is farther from the point of suspension will move down) and the definition of center of gravity (which implies that a body suspended at its center of gravity will equilibrate). It is thus as a consequence of these two physical facts (depending on the property of heaviness) that we can give a first mathematical determination of a center of gravity: that the common center of two equal bodies is located *in the middle* of the line connecting their centers of gravity. Of course, this is still a rather meager result, and the crucial question raised by Mach concerns how we can move from this symmetrical situation to the asymmetrical situation treated in the law of the lever. Also this question is already taken up by Guidobaldo in his comments on this proposition, however, because it is here that he first expounds on what he takes to be the "argumentandi modus," that is, "huius scientiae maximè proprius" (Monte 1588, 44).

Guidobaldo notes that Archimedes makes a very important move in considering the center of gravity of the magnitude *composed* from the two equal magnitudes. Moreover, and this is absolutely crucial, he stresses that since this is one magnitude, it also has one unique center of gravity—and this must be completely independent of the form of the composing magnitudes (Monte 1588, 43; Van Dyck 2006, 376–381)! But as can be seen from Guidobaldo's comments in a scholium preceding the actual proof of the law of the lever, this insight is enough to undercut Mach's criticism of Archimedes's proof (Monte 1588, 55–58). Remember that Guidobaldo had already interpreted the definition of a body's center of gravity by stating that it is in this point that the tendency toward motion of the body is concentrated. We can thus validly assume that the equilibrium that subsists between two bodies will not be disturbed if we replace one of these bodies with two equal bodies, both half its weight, and placed such that their centers of gravity are equally far from its center of gravity—because it follows from the fourth proposition (quoted above) and the definition of center of gravity that both situations are completely equivalent with respect to the physical causes determining the system's equilibrium (same total weight concentrated in the same place).

That this really undercuts Mach's criticism is clear from the fact that at this point we still have no clue about the actual form of the dependency on the distance from the fulcrum at which a weight in equilibrium is suspended.[21] The form of this dependency in no way entered into the argument securing the validity of the replacement, contrary to what is claimed in Mach's analysis. But the validity of the replacement is indeed enough to prove the law of the lever, starting from the fifth proposition, which is basically an extension of the situation described in the fourth proposition where we now consider an arbitrary number of equal magnitudes placed at pair-wise equal distances. The proof of the law of the lever comes down to showing that if the weights of two magnitudes are inversely as the distances from which they are suspended, then these weights can be distributed over

[21] That there must be some kind of dependency is implied by the second postulate, stating that if two equal weights are suspended at unequal distances, the one farther from the point of suspension will go down.

different smaller magnitudes along the line connecting the original magnitudes' centers of gravity in such a way that it follows directly from this fifth proposition that the common center of gravity of all these smaller magnitudes taken together—and *thus* also of the original magnitudes—coincides with the point of suspension (i.e., we can transform the asymmetric case into a symmetric one). This proof involves two essential ingredients: purely geometrical facts about the relations holding between the distances and the smaller magnitudes in which the original magnitudes are divided (facts which show that it is possible to distribute the weights along the line in such a way that the conditions of the fifth proposition will be satisfied); and the assumption that there is a *mechanical equivalence* between the original situation and the one in which the weights are divided and distributed along the line according to the scheme first made clear in the fourth proposition (but now extended to an arbitrary number of parts).

Guidobaldo's care in laying out the conditions which underwrite the validity of this proof is brought out nicely in some of the editorial interpolations which he interjects in the Archimedean text of the proof (interpolations which go to great lengths, but which, with Guidobaldo's characteristic scruples, are clearly marked by using a different typography).[22] After having shown that the weight of the magnitudes A and B can be distributed along the magnitudes STVX and ZM in such a way that the latter's respective centers of gravity will be in E and D, he states the following (Guidobaldo's interpolations are in italics):

> But *magnitudes STVX are equal to magnitude A, & ZM to B, thus* magnitude A is *as it were [tanquam]* placed at E, and B at D; *for certainly the magnitude A placed at E will behave the same way as do the magnitudes STVX; and B will have the same behaviour at D as the magnitudes ZM*. (Monte 1588, 63)

Guidobaldo thus tries to lay bare the "argumentandi modos" (Monte 1588, 55) of Archimedes, by explicitly stating the condition of mechanical equivalence on which the proof is based. It is especially important to notice the care with which he distinguishes between simply identifying the magnitudes and identifying their mechanical effects, by introducing the important qualifier "tanquam," which is absent in the original text.

It should be sufficiently clear by now why the category of the subalternate sciences can offer no insight into the fundaments of Archimedes's science. A close analysis of the crucial proofs makes clear that we must consider the subject matter as *simultaneously* physical and mathematical. Both the mathematical

[22] The Latin text of Archimedes used by Guidobaldo is taken from the 1544 Basel edition, but with some minor terminological changes (e.g., a consistent use of "aequeponderare," whereas the Basel text also uses the expression "aequaliter ponderare").

properties of magnitudes and the physical equivalence between different situations enter critically in the proof of the law of the lever. Guidobaldo's neglect of this category can thus be further explained by the deep insight that he had into the nature of the Archimedean proof procedure. It moreover shows that he indeed had very good reasons to prefer the characterization of the nature of mechanical demonstrations as given by Pappus.

1.6 Some Perspectives on the Problem of Mathematization and the New Philosophy of Nature

Let me now try to draw together the two main lines of argument from the preceding sections, and then offer some reflections on the significance of Guidobaldo's analysis of the Archimedean proof procedure. To start with, I argued that, at least in some possible interpretations, there exists a potential tension between mechanics being a science of *praeter naturam* effects and it being a science of essentially mathematical demonstrations. We should now be able to see clearly how Guidobaldo's *Paraphrasis* offers an essentially different perspective: it is exactly because in mechanics we exploit a *natural property* ("art imitates nature") that the mathematization of its basic properties is possible (the essential role of the natural properties of center of gravity in validating the law of the lever). The concept of center of gravity thus plays a double role in founding the science of mechanics: it provides the artificial effects with a well-defined ontological place in the Aristotelian cosmos; and in doing so it simultaneously allows the epistemological grounding of the law of the lever.

The first role of the concept can be adduced as further proof that Guidobaldo considered it important to inscribe the mathematical science of mechanics within a broad Aristotelian framework, a concern which is also further testified by his attention to the problems regarding the ontological status of the objects of Archimedes's science. Yet we have also seen his neglect of some of the epistemological worries which arise for applied mathematical theories; most crucially: at no point does he pay any attention to the requirement of homogeneity of demonstrations. As I have tried to show, the latter fact can be perfectly well understood if we see how such a focus would make it very hard to properly understand Archimedes's arguments for the law of the lever. Still, it does point to the fact that Guidobaldo, with all his due respect to Aristotle and the Aristotelians, actually might have helped in sowing the seeds of the demise of the Aristotelian philosophy of nature. The concept of center of gravity could find a comfortable place in the Aristotelian cosmos, but the way of exploiting its consequences in constructing a mathematical theory threatens to violate some crucial Aristotelian epistemological constraints. And this violation would actually lead to the progressive dismantling of the Aristotelian cosmos in the hands of people like Galileo. An important factor in this process was the insight that the science of mechanics actually offered an implicit theory of matter; and this insight, I would argue, is prepared in important ways by Guidobaldo's analysis of Archimedes's particular way of considering his subject matter.

So, what could Galileo have learned from the work of people like Guidobaldo? Not only that *it was possible* to give mathematical explanations concerning physical phenomena, but also something about *what made this possible*.

Maybe the most interesting way to think of Guidobaldo's commentary is thus as an analysis of the conditions under which mathematical principles can be considered to be true of physical things. The apparent necessity of offering such an analysis immediately shows that a purely empirical approach is not considered sufficient. This of course is perfectly understandable, as the mathematical principles state very precise relations which can only be approximated in reality. Moreover, this latter fact implies that we will also be confronted with apparent counterexamples, which implies that we must have further grounds to argue that the latter really are indeed only apparent and that their divergence from the ideal case must be ascribed to disturbances and the like. That this is also Guidobaldo's attitude is testified to by an interesting remark in a letter to Giacomo Contarini in which he stresses that although the addition of a small weight does not set an equilibrated equal-arms balance in motion, this does not render the balance false.[23] This betrays the role played by rational argumentation over and above direct empirical information: we know that this aberrant situation (equilibrium for unequal weights) must be due to impediments such as friction, because we have the rational guarantee that the true cause of equilibrium is equality in weight.

Although a purely empirical approach will thus not suffice to lay the foundations of a mathematical science, it is still important to stress that these foundations do crucially involve empirical input. In the case of Archimedes's law of the lever, this input consists in the essential properties of a body's center of gravity, includ-

[23] "La materia fa qualche resistenza […] la qual [materia] vuol la parte sua ancor lei, e quanto sono più grandi in materia tanto più resiste, sì come si prova tutto il giorno nelle libre che, per picole e giuste che le siano e che habbino pesi da tutte due le bande eguali e giusti, non di meno a un di loro se gli potrà metter sopra et aggiunger un peso di tanto poco momento, come un minimo pezzolino di carta che la bilancia starà senza andar giù da detta parte, né per questo la bilancia sarà falsa; dove è da considerare che la resistanza che fa la materia lo fa quando si hanno da mover i pesi e non quando se hanno da sostenere solamente, perché all'hora l'instrumento non si move né gira; e con queste considerationi la troverà sempre che l'esperienza e la demonstrazione andaranno sempre insieme" (Gamba and Montebelli 1988, 76). The context of this remark is Guidobaldo's claim that the rational principles which hold for a balance in rest can not simply be extended to the balance in motion. I will not discuss this issue further, since it demands, as already mentioned, a thorough treatment in its own right.

ing the mere fact of its existence. As Guidobaldo's own discussions make clear, the essential property is that of what we now call indifferent equilibrium (the fact that a body suspended at its center of gravity will always remain in equilibrium, no matter what its orientation with respect to the horizontal). It is this property, crucially exploited in the analysis of Archimedes's proof, that (pre-emptively) undercuts Mach's criticism: if the center of gravity were not a point of indifferent equilibrium then the form of a body would matter, and the crucial transformations could not be effected in the proof. It is thus very significant that Guidobaldo already opened his *Mechanicorum liber* with a long discussion on the possibility and necessity of indifferent equilibrium. This long passage by Guidobaldo has not always fared well among historians of science and has often been badly misinterpreted.[24] His main goal is to discredit the followers of Jordanus, such as Tartaglia, exactly because they had denied the possibility of indifferent equilibrium. Moreover, he introduces the ill-fated idea that the tendencies to move downwards of bodies must converge at the center of the earth in this same context. That such convergence reflects the true physical situation is something on which he agreed with his opponents, but he shows in detail that it invalidates their explanatory scheme whereas he can accommodate the fact. The often repeated judgement that Guidobaldo denounced the ideas of Jordanus out of a misplaced homage to ancient authors (and a consequent rejection of medieval writers), and because he held on to an idea of absolute mathematical rigor, where this latter aspect would be proven by his insistence on the convergence of the lines of descent, is thus very much mistaken. The at first sight convoluted discussion is directed toward one main goal: showing that it is only the notion of center of gravity that allows one to build the science of mechanics from its very foundations. It is thus also of great significance that Guidobaldo, in a passage added in the Italian translation of his *Mechanicorum liber*, claims to have been able to construct a balance that exhibits indifferent equilibrium (Monte 1581, 28v).[25] This proves empirically that bodies do indeed have a point situated within them that shows the required property, contrary to Jordanus's misguided arguments.

The rigor that Guidobaldo searches is thus not absolute mathematical rigor, which would describe the empirical world in full and hideous detail, but the rigor of any well-founded applied mathematical science. And to ground such a science, one has to select—and possibly stabilize experimentally (e.g., by building a balance that shows indifferent equilibrium)—those properties of the empirical world that can be linked with fruitful mathematical demonstrations. This linkage then requires a second component next to this empirical underpinning. As again

[24]For more detailed discussions, see (Van Dyck 2006; Bertoloni Meli 2006).

[25]This text is in the voice of Pigafetta, but Gianni Micheli (1995, 163–167) has published a letter of Guidobaldo to Pigafetta showing that it is actually due to the former.

shown by Guidobaldo's analysis of the Archimedean demonstrations, the empirical information must be processed in a specific type of conceptual argumentation before mathematical consequences can be drawn from it. We have indeed seen how Guidobaldo takes much care in explaining that the Archimedean proof rests on the device of replacing a body with another body having the same mechanical effect.

At one point, Guidobaldo uses a tantalizing choice of words to express that this relation holds when he speaks about the fact that two bodies, which are suspended at their common center of gravity, are "aequipollent" (Monte 1588, 45). This is a term which in the first place expresses the simple fact that the bodies have equal power, but it was also a term with a well-engrained technical meaning within medieval logic, where it expresses something like truth valued equivalence because of the syntactic features of language.[26] We should of course not make too much out of Guidobaldo's rather casual use of this term in just one place, but even then it offers us a glimpse of things to come. The new mathematical science of nature, which would be developed from Galileo onwards, can be seen as an attempt to systematize these relations of causal equivalence by introducing concepts to denote exactly these cases. These can be transformed into each other without altering the effect, thus actually allowing the construction of a logic that is supposed to do justice to the syntax of the world. A prime example, directly linked to Guidobaldo's use of "aequipollence," is of course the introduction of the concept of "momento" to express the equivalence holding in the case of bodies in equilibrium on a balance.

It is precisely the assumed validity of this kind of substitution that also makes clear in what sense the science of mechanics implicitly offered a new philosophy of matter. Assuming bodies to be equivalent and thus substitutable for each other, on account of no more than the fact that they have the same mechanical effect, cuts across most of the Aristotelian categories for judging the identity of objects. Most importantly, according to Archimedes (as interpreted by Guidobaldo) the mere fact of adding a rigid connection between two bodies already turns them into a body having a natural tendency—as expressed by the center of gravity of the composed body. Accordingly, it is not hard to see how his study of Archimedes's science (possibly with the guidance of Guidobaldo) might have convinced someone like Galileo of the fact that we had to do away with all consideration of substantial forms and the like if we wanted to fruitfully analyze the causal structure of the world, and that the new mathematical philosophy of nature should accord-

[26] To quote from a contemporary of Guidobaldo, who moreover is especially well known because of his mathematical and mechanical work, Maurolico gives the following definition: "Aequipollentia est inter propositiones aequivalentia ut essent unum et idem significantes: ut 'quoddam corpus non est animal' et 'non omne corpus est animal' aequipollent" (§ 74 of *Dialectica Maurolyci*, electronic edition on the webpage of the Maurolico project: http://maurolico.free.fr/introen.htm).

ingly start from matter as something that is purely characterized by its mechanical effects.

So finally, what does this tell us about sixteenth-century mechanics as part of a practical tradition of "subalternate sciences"? Let me first go back to Biener's paper in which it is argued that Galileo's first new science in the *Discorsi* "does in fact fit the structure of arguments in the subalternate sciences" (Biener 2004, 281). According to his convincing analysis, the *First Day* establishes that all en-mattered bodies posses certain properties that can be described mathematically, and the second day (which thus constitutes the proper subalternate science) shows that these mathematical properties entail additional mathematical properties (relevant for establishing facts about the fracture of en-mattered bodies). This analysis throws much light on the apparently confusing structure of the arguments in the *First Day*; and Biener's point that this mode of double argumentation must be placed in some kind of practical tradition can indeed be significantly strengthened by comparing it to the relation between Guidobaldo's *Paraphrasis* and his *Mechanicorum liber*: the first establishes that all equilibrium situations can be described mathematically, and the latter exploits this description to demonstrate further mathematical facts which are relevant for establishing facts about the multiplication of force in all simple machines (again implying that only the *Mechanicorum liber* would constitute a subalternate science). But, if this is the general outlook on the structure of applied mathematical sciences that Galileo inherited from the practical tradition, my analysis points to the fact that in this tradition itself there was already considerable attention paid to the thorny question of how to establish a sufficiently rich and fruitful minor premise (the subject of Guidobaldo's *Paraphrasis*, and later of Galileo's *First Day*). It is here that the true challenge arose for the establishment of mathematical sciences that validly capture physical phenomena; and it is here that the philosophical tradition remained silent, and moreover in this silence obscured the most important insight that Guidobaldo already explicated: that this challenge could only be met by clinching a "modo argumentando" that was truly *sui generis*, being simultaneously mathematical and physical.[27]

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[27]One could plausibly argue that the semantic shift toward "mixed" as the name for the category of applied mathematics was at least helped by this insight.

the issues treated here. As always, I would like to acknowledge the fact that this research would not have been possible without the wonderful resources provided by the Archimedes project (http://archimedes.mpiwg-berlin.mpg.de) which provides the full texts of all the early modern books discussed in this paper.

References

Aristotle (1930). Physica. De caelo. De generatione et corruptione. In: *The Works of Aristotle*. II. Oxford: Clarendon.

Baldi, B. (1621). *In mechanica Aristotelis problemata exercitationes: adiecta succinta narratione de autoris vita et scriptis*. Mainz: Typis & Sumptibus Viduae Ioannis Albini.

Bertoloni Meli, D. (1992). Guidobaldo dal Monte and the Archimedean Revival. *Nuncius* 7:3–34.

— (2006). *Thinking with Objects. The Transformation of Mechanics in the Seventeenth Century*. Baltimore: John Hopkins University Press.

Biener, Z. (2004). Galileo's First New Science: The Science of Matter. *Perspectives on Science* 12:262–287.

Cicero, M. T. (1971). *Tusculan Disputations*. Cambridge, Mass.: Harvard University Press.

Dear, P. (1995). *Discipline & Experience*. Chicago: University of Chicago Press.

Dijksterhuis, E. J. (1956). *Archimedes*. Kopenhagen: Ejnar Munksgaard.

Drake, S. and I. E. Drabkin (1969). *Mechanics in Sixteenth-Century Italy*. Madison: University of Wisconsin Press.

Feldhay, R. (1999). The Cultural Field of Jesuit Science. In: *The Jesuits: Cultures, Sciences, and the Arts 1540–1773*. Ed. by J. O'Malley. Toronto: University of Toronto Press, 107–131.

Festa, E. and S. Roux (2001). Le 'παρὰ φύσιν' et l'imitation de la nature dans quelques commentaires du prologue des Questions mécaniques. In: *Largo Campo di Filosofare*. Ed. by J. Montesinos and C. Solís. La Orotava: Fundación Canaria Orotava de Historia de la Ciencia, 237–253.

Gamba, E. and V. Montebelli (1988). *Le scienze a Urbino nel tardo Rinascimento*. Urbino: QuattroVenti.

Glare, P. G. W., ed. (1996). *Oxford Latin Dictionary*. Oxford: Clarendon Press.

Laird, W. R. (1983). *The Scientiae Mediae in Medieval Commentaries on Aristotle's Posterior Analytic*. PhD thesis. Unpublished Ph. D. Dissertation, Toronto.

— (1986). The Scope of Renaissance Mechanics. *Osiris* 2, s. II:43–68.

Laird, W. R. (1997). Galileo and the Mixed Sciences. In: *Method and Order in Renaissance Philosophy of Nature*. Ed. by D. A. Di Liscia, E. Kessler, C. Methuen. Aldershot: Ashgate Publishing Limited, 253–270.

Lennox, J. G. (1986). Aristotle, Galileo and "Mixed sciences". In: *Reinterpreting Galileo*. Ed. by W. A. Wallace. Washington (D.C.): Catholic University of America Press, 29–52.

Machamer, P. (1978). Galileo and the Causes. In: *New Perspectives on Galileo*. Ed. by R. E. Butts and J. C. Pitt. Dordrecht: Springer, 161–180.

Mach, E. (1960). *The Science of Mechanics*. La Salle (Ill.): Open Court Publishing.

Maurolico, F. (1613). *Problemata mechanica*. Messina: Brea.

Micheli, G. (1995). *Le origini del concetto di macchina*. Firenze: Leo S. Olschki.

Monantheuil, H. (1599). *Aristotelis mechanica graeca, emendate, latina facta, & commentariis illustrate*. Paris: Ieremiam Perier.

Monte, Guidobaldo del (1577). *Mechanicorum liber*. Pesaro: Hieronymum Concordiam.

— (1581). *Le mechaniche dell'illustriss. sig. Guido Ubaldo de' Marchesi del Monte: Tradotte in volgare dal sig. Filippo Pigafetta*. Venezia: Francesco di Franceschi Sanese.

— (1588). *In duos Archimedis aequeponderantium libros paraphrasis scholijs illustrata*. Pesaro: Hieronymum Concordiam.

Napolitani, P. D. (1982). Metodo e statica in Valerio. *Bollettino di storia delle scienze matematiche* 2:3–86.

Pappus, A. (1660 [1588]). *Mathematicae collectiones a Federico Commandino Urbinate in Latinum conversae, & commentarijs illustratae*. Bologna: HH. de Duccijs.

Piccolomini, A. (1565). *In mechanicas quaestiones Aristotelis, paraphrasis paulo quidem plenior. [...] Eiusdem commentarium de certitudine mathematicarum disciplinarum*. Venezia: Curzio Troiano Navo.

Popplow, M. (1998). *Neu, nützlich und erfindungsreich: Die Idealisierung von Technik in der Frühen Neuzeit*. Münster: Waxmann.

Tartaglia, N. (1546). *Quesiti et inventioni diverse*. Repr. in facsimile Brescia: Ateneo di Brescia, 1959. Venezia: Venturino Ruffinelli.

Vailati, G. (1987). *La dimostrazione del principio della leva data da Archimede nel libro primo sull'equilibrio delle figure piane*. In: *Scritti*. Ed. by M. Quaranta. Vol. 2. Bologna: Arnaldo Forni, 220–225.

Van Dyck, M. (2006). Gravitating Towards Stability: Guidobaldo's Aristotelian-Archimedean Synthesis. *History of Science* 44:373–407.

Wallace, W. A. (1984). *Galileo and His Sources*. Princeton, N. J.: Princeton University Press.