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Entanglement as an Element-of-Reality

Philip Walther

Abstract. Entanglement—according to Schrödinger (1935) the essential property of quantum mechanics—teaches us that the properties of individual quantum systems cannot be considered to be (local) elements of physical reality before and independent of observation. Yet it is a widespread point of view that the way the observations on, say, two particles are correlated, i.e. the specific type of their entanglement, can still be considered as a property of the physical world. Here I discuss a previous experiment (Walther et al., 2006) showing that this is explicitly not the case. The correlations between a single particle property, the polarization state of a photon, and a joint property of two particles, the entangled state of a photon pair in a three-photon entangled state, have been measured. It is shown that the correlations between these properties can obey a cosine relation in direct analogy with the polarization correlations in one of the triplet Bell states (Bell, 1964). The cosine correlations between the polarization and entangled state measurements are too strong for any local-realistic explanation and are experimentally exploited to violate a Clauser-Horne-Shimony-Holt (CHSH) Bell inequality (Bell, 1964; Clauser et al., 1969). Thus, entanglement itself can be an entangled property leading to the notion of entangled entanglement.

1 Introduction

In general, quantum mechanics only makes probabilistic predictions for individual events. Can one go beyond quantum mechanics in this respect? More than seventy years ago, in 1935, Einstein, Podolsky and Rosen (EPR) argued that quantum theory could not possibly be complete (Einstein et al., 1935). They showed that one could infer perfectly complementary properties, like position and momentum of an individual particle, by performing a corresponding measurement on the distant particle that is quantum-mechanically entangled with the first one. Based firmly on plausible assumptions about locality, realism, and theoretical

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completeness, they further argued that quantum states cannot be a complete description of physical reality, but rather give only a statistical one of an ensemble of intrinsically different quantum systems. While at the time, Bohr (1935) famously argued against EPRis conclusions, in particular against their notion of "reality" as assuming the systems have intrinsic properties independently of whether they are observed or not, it was not until almost 30 years later that the EPR program could be formulated in terms of an experimentally-testable prediction. I am, of course, referring to the landmark discovery of John Bell (1964) that EPRis premises of locality and realism put measurable limits on the strength of correlations between outcomes of remote measurements on a pair of systems. These limits are known as Bell inequalities and quantum mechanics does not satisfy them.

Since Bell's initial discovery, a large volume of theoretical and experimental work has been devoted to this subject. Experimental violations of Bell inequalities have been demonstrated using pairs of polarization-entangled photons (Freedman and Clauser, 1972; Fry and Thompson, 1976; Aspect et al., 1982; Ou and Mandel, 1988; Shih and Alley, 1988; Weihs et al., 1998), even under strict Einstein locality requirement, using other photonic degrees of freedom such as energy-time (Tapster et al., 1994; Tittel et al., 1998) and angular momentum (Vaziri et al., 2002), trapped ions (Rowe et al., 2001), and even neutron systems (Hasegawa et al., 2004). Multiphoton entanglement experiments have been performed demonstrating all-versus-nothing arguments against local realism (Pan et al., 2000) by exploiting so-called Greenberger-Horne-Zeilinger (GHZ) states (Greenberger et al., 1989), where single measurement outcomes can be incompatible with local realistic models. Aside from outstanding loopholes, which have not all been closed simultaneously in a single experiment (Weihs et al., 1998; Rowe et al., 2001), these experiments all but rule out the possibility of local realistic theories. However, common to all previous Bell experiments, regardless of the implementation, is that the measured degrees of freedom corresponded to properties of individual systems. Entanglement itself, as a property of the composite systems, was usually considered an objective property.

The experiment discussed in the following, however, demonstrated the first example of a Bell-inequality violation where an entangled state itself qualifies as an EPR element of reality. Specifically, a measurement of the single particle at Alice's side defines the relational property between the two other particles, without defining their single-particle properties. Therefore, only the joint state of the two qubits at Bob's side is an element of reality. The correlations between the polarization state of one photon and the entangled state of another two are experimentally demonstrated to violate the Bell inequality. This shows that entanglement itself can be entangled. The notion that entanglement itself can be an entangled property was originally proposed in the context of (Zeilinger et al., 1992; Krenn and Zeilinger, 1996).

2 An Experiment on Entangled Entanglement

In Figure 1, a schematic for the experiment is shown in which three photons are prepared in an entangled state

$$|\Phi^{-}\rangle_{1,2,3} = \frac{1}{\sqrt{2}} \left(|H\rangle_{1} |\phi^{-}\rangle_{2,3} - |V\rangle_{1} |\psi^{+}\rangle_{2,3} \right), \tag{1}$$

where the subscripts label different photons, the kets $|H\rangle_1$ and $|V\rangle_2$ represent states of horizontal and vertical polarization, respectively, of photon 1 and $|\psi^+\rangle_{23} = 1/\sqrt{2}(|H\rangle_2 |V\rangle_3 + |V\rangle_2 |H\rangle_3$ and $|\phi^-\rangle_{12} = 1/\sqrt{2}$ $(|H\rangle_1 |H\rangle_2 + |V\rangle_1 |V\rangle_2$ represent two (out of four possible) so-called Bell-states (maximally entangled states) of photons 2 and 3. Since the entangled state of photons 2 and 3 is entangled with the polarization state of photon 1, the state in Eq. (1) can be referred to as entangled entanglement. Photon 1 is moving freely in one direction to Alice, while the photons 2 and 3 are moving into the opposite direction to Bob. Aliceís photon 1 is now subjected to a polarization measurement along the axis θ_1 . For simplicity, the settings are restricted to the linear polarization measurement, i.e., θ_1 lies within the x-y plane of the Poincaré sphere. If the polarization is found to be parallel to the axis θ_1 (outcome +1), the photon will be projected onto the state $|H'\rangle_1 = \cos \theta_1 |H\rangle_1 + \sin \theta_1 |V\rangle_1$, or when to be found perpendicular (outcome -1), it will be projected onto the state $|V'\rangle_1 = -\sin\theta_1 |H\rangle_1 + \cos\theta_1 |V\rangle_1$. Photons 2 and 3 at Bobís side are subjected to a specific joint measurement that can also only result in two different outcomes. In relation to the experiment, photons 2 and 3 are labelled as B and D, respectively, due to being emitted into the spatial mode B and D (Figure 2). Bob's measurement setting is denoted by the angle θ_2 . The measurement will project the two photons onto either the state $|\phi^{-\prime}\rangle_{B,D} = \cos\theta_2 |\phi^{-\prime}\rangle_{B,D} + \sin\theta_2 |\psi^{+\prime}\rangle_{B,D}$ (outcome +1) or $|\psi^{+'}\rangle_{B,D} = -\sin\theta_2 |\phi^{-}\rangle_{B,D} + \cos\theta_2 |\psi^{+}\rangle_{B,D}$ (outcome -1). The outcome +1 will be identified by joint registration of photons 2 & 3 at the pairs of detectors, (1 and 2) or (3 and 4), while the outcome -1 will be identified

Philip Walther

by firing of pairs of detectors (1 and 3) or (2 and 4). When Alice and Bob choose the orientations θ_1 and θ_2 of their measurement apparatuses the initial state transforms to

$$\begin{split} \left| \Phi^{-} \right\rangle_{1,B,D} &= \cos(\theta_{1} + \theta_{2}) \frac{1}{\sqrt{2}} \left(\left| H \right\rangle_{1} \left| \phi^{-} \right\rangle_{B,D} - \left| V \right\rangle_{1} \left| \psi^{+} \right\rangle_{B,D} \right) \\ &+ \sin(\theta_{1} + \theta_{2}) \frac{1}{\sqrt{2}} \left(\left| V \right\rangle_{1} \left| \phi^{-} \right\rangle_{B,D} - \left| H \right\rangle_{1} \left| \psi^{+} \right\rangle_{B,D} \right). \end{split}$$
(2)

The quantum state in Eq. (1) has the remarkable property that it is the same for any choice of local settings θ_1 and θ_2 such that $\theta_1 = -\theta_2$, i.e., it is invariant under this set of locally unitary transformations. This entails *perfect correlations*: if polarization along θ_1 is found to be +1 (-1) for photon 1, then with certainty the result of the measurement for setting θ_2 will be found to be +1 (-1) for photons 2 and 3, and vice versa. Because of the perfect correlations, the result of measuring any entangled state $\cos \theta_2 |\phi^-\rangle_{B,D} + \sin \theta_2 |\psi^+\rangle_{B,D}$ or $-\sin \theta_2 |\phi^-\rangle_{B,D} + \cos \theta_2 |\psi^+\rangle_{B,D}$ can be predicted with certainty by previously choosing to measure the polarization of photon 1 along the axis $\theta_1 = -\theta_2$. By locality (in EPR's words):

Since at the time of measurement the two systems no longer interact, no real change can take place in the second system in consequence of anything that may be done to the first system,

the measurement performed on photon 1 (photons 2 and 3) can cause no real change in photons 2 and 3 (photon 1). Thus, by the premise about reality (in EPR's words):

If, without in any way disturbing a system, we can predict with certainty (i.e. with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical reality,

the entangled states of photons 2 and 3 are elements of reality for any θ_2 (and similarly for photon 1 and its polarization along θ_1). Remarkably, the individual properties of either photon 2 or 3 are not well-defined, as individual detection events at detectors 1, 2, 3, and 4 are random and cannot be predicted by previously choosing to measure a property of photon 1. Therefore, the EPR elements of

reality for entangled states of photons 2 and 3 may exist even without existence of these elements for their individual properties.

In the following, I will demonstrate that the conjunction of EPR's propositions, which lead to the establishment of entangled states as elements of reality, is in conflict with the quantum-mechanical prediction. This incompatibility will be shown by deriving CHSH Bell inequality [4] for correlations between individual properties of photon 1 and joint properties of photons 2 and 3 from EPR premises and experimental demonstration of their violation by quantum mechanical predictions.

While any Bell state can be converted into any other Bell state by only singlequbit rotations on one of its constituents (Mattle et al., 1996), the argument is constructed by using a specific subset of two of the Bell states, $|\psi^+\rangle_{2,3}$ and $|\phi^-\rangle_{2,3}$, since they are coherently mixed through the polarization rotation introduced by a half-wave plate (HWP), which makes such an experiment feasible. Using only this HWP, projective measurements onto maximally entangled states of the form $\cos \theta_2 |\phi^-\rangle_{2,3} + \sin \theta_2 |\psi^+\rangle_{2,3}$ at Bobís side can be controlled. For consistency throughout this paper, the angle θ has been adopted to mean the rotation of a polarization in real space. Thus the same polarization rotation on the sphere is $2\theta_2$ and that rotation is induced by an HWP which is itself rotated by only $\theta_2/2$.

The experimental setup is explicitly explained in (Kwiat et al., 1995): The three-photon state is created using a pulsed ultraviolet laser (pulse duration 200 fs, repetition rate 76 MHz), which makes two passes through a type-II phasematched β -barium borate (BBO) nonlinear crystal (Mattle et al., 1996), in such a way that it emits highly polarization-entangled photon pairs into the modes A & B and C & D (Figure 2). Transverse and longitudinal walk-off effects are compensated using an HWP and an extra BBO crystal in each of modes A through D. By additionally rotating the polarization of one photon in each pair with additional HWPs and tilting the compensation crystals, any of the four Bell states can be produced in the forward and backward direction. The source is aligned to produce the Bell state, $|\phi^+\rangle$, on each pass of the pump. Photons are detected using fibre-coupled single-photon counting modules and spectrally and spatially filtered using 3nm bandwidth filters and single-mode optical fibres. While classically correlated states cannot be correlated at the same time in complementary bases, the quality of entanglement is confirmed by the measured visibilities of each generated photon pair, which exceeded 95% in the H/V basis and 94% in the complementary $|\pm\rangle = 1/\sqrt{2}(|H\rangle \pm |V\rangle)$ basis.

168

Bell pairs contain only two-particle entanglement. To entangle them further, one photon from each pair needs to be superimposed: those in modes A and C, on a polarizing beamsplitter (PBS1). Provided those photons overlap at the beamsplitter and emerge from different output ports, a four-photon GHZ state is generated (Mattle et al., 1996) $|\Psi\rangle$ $1/\sqrt{2} (|H\rangle_B |H\rangle_D |H\rangle_1 |H\rangle_T + |V\rangle_B |V\rangle_D |V\rangle_1 |V\rangle_T$. The PBS is an optical device that transmits horizontally-polarized photons and reflects verticallypolarized photons. The PBS implements a two-qubit parity check: if two photons enter the PBS from the two different input ports, then they must have the same polarization in the H/V basis in order to pass to the two different output ports. Then, rotations incurred in guarter-wave plates (QWP) and the subsequent projection of the trigger photon in mode T onto $|H\rangle_T$ reduces the four-particle GHZ state to the desired three-photon entangled state $\left|\Phi^{-}\right\rangle_{1,B,D} = \frac{1}{\sqrt{2}} \left(\left|H\right\rangle_{1} \left|\phi^{-}\right\rangle_{B,D} - \left|V\right\rangle_{1} \left|\psi^{+}\right\rangle_{B,D}\right).$

The polarization of single photons can easily be measured by using linear polarizers. As is common in Bell experiments, the angle, θ_1 , defines the state on which the linear polarizers projects. In this work, for Bobis measurement, a Bellstate analyzer based on a PBS (Pan and Zeilinger, 1998) is used. By performing a check that the parity of the photons is even, the PBS acts as a $|\phi^{\pm}\rangle$ -subspace filter. The two Bell states in this subspace, $|\phi^{+}\rangle$ and $|\phi^{-}\rangle$, have opposite correlations in the $|\pm\rangle$ basis and can easily be distinguished using a pair of linear polarizers. By orienting those linear polarizers so that one is along the $|+\rangle$ direction and the other along the $|-\rangle$ direction, a projective measurement onto $|\phi^{-}\rangle$ is completed. Since an HWP in mode *B* can interconvert $|\phi^{-}\rangle$ and $|\psi^{+}\rangle$ in a controllable way, Alice can choose her projective measurement before her PBS is set to an angle $\theta_2/2$. This is directly analogous to the projections onto the polarization state.

Correlation measurements were carried out by rotating Aliceis polarizer angle, θ_1 , in 30° steps while Bob's HWP was kept fixed at $\theta_2/2 = 0^\circ$ or 22.5°. Four-fold coincidence counts at each setting were measured for 1800 seconds. These data are shown in Figure 3. The count rates follow the expected relation $N(\theta_1, \theta_2) \propto \cos^2(\theta_1 + \theta_2)$ in analogy with the expected rates from the standard two-particle Bell experiment. The experimentally obtained data have visibilities of $(78 \pm 2)\%$ in the H/V-basis and $(83 \pm 2)\%$ in the $|\pm\rangle$ basis. Both of these visibilities surpass the crucial limit of ~ 71% which, in the presence of white noise, is the threshold for demonstrating a violation of the CHSH-Bell inequality. Thus, for the proper choices of measurement settings it is expected that the entangled entangled state should be able to demonstrate a conflict with local realism using Aliceís polarization state and Bobís maximally-entangled state.

For the state, $|\Phi^-\rangle_{1,B,D}$, the expectation value for the correlations between a polarization measurement at Bob and a maximally-entangled state measurement at Alice is $E(\theta_1, \theta_2) = \cos [2(\theta_1 + \theta_2)]$. The correlation can be expressed in terms of experimentally-measurable counting rates using the relation

$$E(\theta_{1},\theta_{2}) = \frac{N(\theta_{1},\theta_{2}) + N(\theta_{1} + \frac{\pi}{2},\theta_{2} + \frac{\pi}{2}) - N(\theta_{1},\theta_{2} + \frac{\pi}{2}) - N(\theta_{1} + \frac{\pi}{2},\theta_{2})}{N(\theta_{1},\theta_{2}) + N(\theta_{1},\theta_{2} + \frac{\pi}{2}) + N(\theta_{1} + \frac{\pi}{2},\theta_{2}) - N(\theta_{1} + \frac{\pi}{2},\theta_{2} + \frac{\pi}{2})}$$
(3)

where $N(\theta_1, \theta_2)$ is the number of coincidence detection events between Alice and Bob with respect to their set of analyzer angles θ_1 and θ_2 . These correlations can be combined to give the CHSH-Bell parameter, $S = |-E(\theta_1, \theta_2) + E(\tilde{\theta}_1, \theta_2) + E(\tilde{\theta}_1, \tilde{\theta}_2)|$, where $S \le 2$ for all local realistic theories. For the settings $\{\theta_1, \tilde{\theta}_1, \theta_2, \tilde{\theta}_2\} = \{0^o, 45^o, 22.5^o, 67.5^o\}$, the correlations calculated from quantum mechanics for our state yields $S = 2\sqrt{2}$. This value violates the CHSH Bell inequality and is therefore incompatible with the assumptions of local realism (Fry and Thompson, 1976).

In the experiment, four-fold coincidence counts at each measurement setting were accumulated for 1800 seconds. Each four-fold coincidence signalled 1) the successful creation of two pairs, 2) the successful entangling operation at PBS1, 3) the reduction of the state to the three photon state onto the requisite state, $|\Phi^-\rangle_{1,B,D} = \frac{1}{\sqrt{2}} \left(|H\rangle_1 |\phi^-\rangle_{B,D} - |V\rangle_1 |\psi^+\rangle_{B,D} \right).$

As is shown in Eq. 3, each correlation is a function of four such data points. The counting rates are shown in Figure 4 for the 16 required measurement settings. These counting rates allow us to calculate the four correlations $E(\theta_1, \theta_2) = 0.69 \pm 0.05$, $E(\theta_1, \tilde{\theta}_2) = -0.61 \pm 0.04$, $E(\tilde{\theta}_1, \theta_2) = -0.58 \pm 0.04$ and $E(\tilde{\theta}_1, \tilde{\theta}_2) = -0.60 \pm 0.04$. Furthermore, those correlations give the experimental Bell parameter, $S = 2.48 \pm 0.09$. This Bell parameter violates the CHSH inequality by 5.6 standard deviations.

3 Conclusion

This year, the Bell inequality turned 47. Since their inception, Bellis inequalities have been the subject of immense theoretical and experimental interest. Initially,

this effort was focused on purely foundational issues, but more recently, this work has grown into the burgeoning field of quantum information. Even with all of this attention to this topic, Bell tests have been considered only using single particle properties. The experimental work discussed here is the first Bell test where this restrictive constraint has been lifted.

This result also shows that the naive realistic view of "particles" being physical entities that can be entangled is too simplistic and narrow as no single particle properties are entangled in the present experiment. Therefore from an information-related point of view it only makes sense to speak about measurement events (detector "clicks") whose statistical correlations may violate limitations imposed by local realism and thus be entangled.

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Figure 1: Schematic for the Bell experiment based on an entangled entangled state. a) A source emits three entangled photons in such a way that one photon is received by Alice and the two other photons by Bob. Alice controls an analyzer that makes measurements of the polarization of her photon. When the photonis polarization is measured to be parallel to orientation, θ_1 , of the analyzer, the measurement outcome is +1 (red light bulb) or -1 (green light bulb) when perpendicular. In contrast, Bob makes projective measurements onto a two-particle entangled state, where again the orientation of the apparatus is defined by the angle, θ_2 . Bobís outcomes are defined as +1, when detectors 1 & 2 (red light bulbs) or 3 & 4 (green light bulbs) are firing, or -1 when detectors 1 & 3 or 2 & 4 are firing. b) When Alice and Bob measure with the same measurement settings, i.e. $\theta_1 = -\theta_2$, they observe perfect correlations, which appear in four possible configurations, given by +1. However, when they measure in a different basis, i.e. $\theta_1 \neq \theta_2$, they will also observe four possible anti-correlations c), given by -1. The correlation measurements with different measurement settings form the basis of a test of local realism using entangled entanglement.



Figure 2: Setup for the experimental realization. A spontaneous parametric down-conversion source emits polarization-entangled photons in the Bell state, $|\phi^+\rangle$, into both the forward pair of modes A & B and backward pair of modes C & D. After superimposing the modes A & C at the polarizing beamsplitter PBS1, passing each mode through a quarter-wave plate (QWP), and projecting the trigger qubit T onto the state $|H\rangle_t$ generates the entangled entangled state $|\Phi^-\rangle_{1,B,D} = \frac{1}{\sqrt{2}} (|H\rangle_1 |\phi^-\rangle_{B,D} - |V\rangle_1 |\psi^+\rangle_{B,D})$. The photon in mode 1 belongs to Alice, who uses a linear polarizer for her single-particle polarization measurements, determined by the angle, θ_1 , of her polarizer. The photons in mode B and D belong to Bob, who uses a modified Bell state analyzer to make projections onto a coherent superposition of $|\phi^-\rangle_{B,D}$ and $|\psi^+\rangle_{B,D}$, where the mixing angle, θ_2 , is determined by the angle, $\theta/2$, of the half-wave plate (HWP) in mode B.



Figure 3: Measured coincidence fringes between Alice and Bob for the entangled entangled state. Bobís half-wave plate was initially set to 0∞ , so that he made fixed projective measurements onto the state $|\phi^-\rangle_{B,D}$. The total number of four-fold coincidence counts measured in 1800 seconds as a function of the angle of Alice's polarizer is shown as solid squares. Fitting the curve to a sinusoid (solid line) yields a visibility of $(78 \pm 2)\%$. Bob then changed his measurement setting to project onto the state $\frac{1}{\sqrt{2}} (|\phi^-\rangle_{B,D} + |\psi^+\rangle_{B,D})$, and the procedure was repeated. The data for these settings are shown as open circles. The sinusoidal fit (dotted line) yields a visibility of $(83 \pm 2)\%$.



Figure 4: Experimental results obtained by measuring correlations for violating a CHSH Bell inequality. The Bell inequality is comprised of 4 correlations, in this case between the polarization state measured by Alice and the entangled states measured by Bob. Each of these correlations in turn can be extracted from 4 coincidence counting rates. The requisite coincidence measurements for the 16 different measurement settings are shown. Each measurement was performed for 1800 seconds. For measurement settings, $\{\theta_1, \theta_2\}$, the axis labels ++, +-, -+, -- refer to the actual settings of $\{\theta_1, \theta_2\}$, $\{\theta_1, \theta_2 + \pi/2\}$, $\{\theta_1 + \pi/2, \theta_2\}$, and $\{\theta_1 + \pi/2, \theta_2 + \pi/2\}$ respectively. These data can be combined to give the Bell parameter $S = 2.48 \pm 0.09$.